

Investigations on power penalty at different spectral width using small signal analysis with higher-order dispersion

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Abstract

In this paper, a modified small signal analysis for power penalty at different spectral width of the light source has been investigated by incorporating the higher-order dispersion parameters. Further, we have analyzed the individual and combined effects of second- and third-order dispersion parameter on power penalty at different spectral line widths. The results have been reported over the range of 10–1000 km by considering the impact of individual and the combined higher-order dispersion terms. It has been observed that by reducing the spectral line width of the light source, the power required at the receiver can be minimized. The power penalty is further reduced if only third-order dispersion parameter is considered. Further, the transmitter distance can be maximized if the spectral width of the light reduces to 100 kHz.

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1. Introduction

Recently, for achieving high-bit-rate transmission single mode fibers are usually used in low fiber-loss-transmission windows (1300–1550 nm). But it was observed that there is another impairment i.e. chromatic dispersion that degrades the overall system performance of an optical communication system. At high-bit rate, the dispersion-induced broadening of short pulses propagating in the fiber causes cross talk between the adjacent time slots as the optical distance increases beyond the dispersion length of the fiber.

Higher-order dispersion terms are the forces destructive of pulse propagation in ultra high-bit rate optical transmission system and cause power penalty in the system. The power penalty in presence of impairment is defined as the increase in signal power required (in dB) maintaining the same bit error rate as in the absence of that impairment. Therefore, in order to realize the high data rates over long distances down the SM fiber, techniques must be found to overcome the pulse spreading and reduce power penalty due to dispersion.

The various methods have been discussed in the past to analyze the propagation of modulated signal produced by lasers through a dispersive medium. Wang et al. [1] investigates the impact of dispersion terms on optical fiber communication systems using small signal analysis. Crognale et al. [2] extended the analysis of the

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Wang by comparing the effect of second- and third-order dispersion term. Peral et al. [3] derived an expression for the large signal theory for the propagation of an optical wave with sinusoidal amplitude and the frequency modulation in the dispersive medium. Peterman et al. [4] discussed the FM–AM conversion for a dispersive optical fiber with respect to binary intensity modulated PCM systems. This work is also limited to the first order dispersion parameter. Further the work reported in [1,2] was extended in [5,6] by presenting an improved analysis for analyzing the influence of the higher-order dispersion on dispersive optical communication system.

In this paper, the effect of spectral width over power penalty under the individual effects of second- and third-order parameter and the combined effect of second- and third-order dispersion parameters for the transmission distance range of 10–1000 km have been reported. The effect of spectral width of light source including second- and third-order dispersion parameters has been observed and explained in Section 2 and the results have been discussed in Section 3.

2. Theory

Let us consider a single mode fiber transmission line and assume that the input field at the fiber input can be given by Aggarwal [9] as

$$E(t) = E_{in}(t) e^{i\omega t}. \quad (1)$$

The input field can be transferred to the output field as

$$E_o(\omega) = E_{out}(\omega) e^{i\omega L} \quad (2)$$

with the slow varying complex field amplitude ‘ $E_o(t)$ ’ at the fiber output, The propagation of a signal can be described in the terms of propagation constant ‘ β ’ by the equation described in term of Fourier transform as

$$E_o(\omega) = E_{in}(\omega) e^{-i\beta L}, \quad (3)$$

where the propagation constant in terms of Taylor series can be expanded as

$$\begin{aligned} \beta &= \beta_o + (\omega - \omega_o) \frac{d\beta}{d\omega} + \frac{1}{2} (\omega - \omega_o)^2 \frac{d^2\beta}{d\omega^2} \\ &+ \frac{1}{6} (\omega - \omega_o)^3 \frac{d^3\beta}{d\omega^3} + \dots \end{aligned} \quad (4)$$

Here, $d\beta/d\omega = \tau$ is the propagation delay per optical length. Now

$$\begin{aligned} \beta &= \beta_o + (\omega - \omega_o)\tau + \frac{1}{2} (\omega - \omega_o)^2 \frac{d\tau}{d\omega} \\ &+ \frac{1}{6} (\omega - \omega_o)^3 \frac{d^2\tau}{d\omega^2} + \dots \end{aligned} \quad (5)$$

where

$$\beta_2 = \text{first-order dispersion} = -\frac{\lambda^2}{2\pi c} \frac{\partial \tau}{\partial \lambda}, \quad (6)$$

$\beta_3 = \text{second-order dispersion}$

$$= \frac{\lambda^2}{(2\pi c)^2} - \left[\lambda^2 \frac{\partial^2 \tau}{\partial \lambda^2} + 2\lambda \frac{\partial \tau}{\partial \lambda} \right]. \quad (7)$$

Thus, by inserting Eq. (5) in Eq. (3), we have

$$E_o(\omega) = E_{in}(\omega) \times e^{-i\beta_o L - iL(\omega - \omega_o)\tau - iL\frac{1}{2}(\omega - \omega_o)^2 \frac{d\tau}{d\omega} - iL\frac{1}{6}(\omega - \omega_o)^3 \frac{d^2\tau}{d\omega^2} + \dots} \quad (8)$$

As reported in [6], we neglect the phase and group delay i.e. $\beta_o L$ and τ because both terms produce only phase delay of the carrier signal and have no influence on the distortion on the signal. We define the following dispersion parameters as

$$F_2 = -\frac{L}{2} \frac{d\tau}{d\omega} = \frac{L}{2} \frac{\lambda^2}{(2\pi c)} \frac{\partial \tau}{\partial \lambda}, \quad (9)$$

$$F_3 = \frac{L}{6} \frac{d^2\tau}{d\omega^2} = \frac{L}{2} \frac{\lambda^2}{(2\pi c)^2} \left[\lambda^2 \frac{\partial^2 \tau}{\partial \lambda^2} + 2\lambda \frac{\partial \tau}{\partial \lambda} \right]. \quad (10)$$

Now, the output equation in terms of Fourier domain can be written by inserting Eqs. (9) and (10) in Eq. (8) as

$$E_o(\omega) = e^{[i(\omega - \omega_o)^2 F_2 - i(\omega - \omega_o)^3 F_3 \dots]} E_{in}(\omega). \quad (11)$$

In time domain

$$\left(i\omega = \frac{\partial}{\partial t} \right), \left((i\omega)^2 = \frac{\partial^2}{\partial t^2} \right) \quad \text{and} \quad \left((i\omega)^3 = \frac{\partial^3}{\partial t^3} \right).$$

Now, Eq. (11) becomes

$$E_o(t) = e^{[-iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^3}{\partial t^3} \dots]} E_{in}(t) e^{i\phi(t)}. \quad (12)$$

Also,

$$E_{in}(t) = \sqrt{P(t)} e^{i\phi(t)}. \quad (13)$$

On inserting Eq. (13) in Eq. (12), we get

$$E_o(t) = e^{[iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^3}{\partial t^3} \dots]} \sqrt{P(t)} e^{i\phi(t)}. \quad (14)$$

But

$$E_{out}(t) = E_{in}(t) + \Delta E(t), \quad (15)$$

where $|\Delta E(t)| = |E_{in}(t)|$.

From Eqs. (14) and (15),

$$\Delta E(t) = \left[e^{(-iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^3}{\partial t^3} \dots)} - 1 \right] \sqrt{P} e^{i\phi(t)}. \quad (16)$$

Also,

$$\begin{aligned} P_{out}(t) &= |E_{in}(t) + \Delta E(t)|^2 \\ &\approx |E_{in}(t)|^2 + 2\Re[E_{in} * (t)\Delta E(t)]. \end{aligned} \quad (17)$$

Insert Eqs. (13) and (15) in Eq. (17), we get

$$P_{\text{out}} = P + 2\mathfrak{N} \times \left[\sqrt{P(t)} e^{-i\phi(t)} \left(e^{(-iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^2}{\partial t^2} + \dots)} - 1 \right) \sqrt{P(t)} e^{i\phi(t)} \right]. \quad (18)$$

Express $e^x = 1 + x + x^2 \dots = 1 + x$, because for PCM transmission, the spectrum due to noise is considered to be narrow. So, we get

$$P_{\text{out}}(t) = P + 2\mathfrak{N} \times \left[\sqrt{P(t)} e^{-i\phi(t)} \left(-iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^2}{\partial t^2} \dots \right) \sqrt{P(t)} e^{i\phi(t)} \right]. \quad (19)$$

This equation can be expressed as

$$P_{\text{out}}(t) = P + \Delta P,$$

where

$$\Delta P = 2\mathfrak{N} \times \left[\sqrt{P(t)} e^{-i\phi(t)} \left(e^{(-iF_2 \frac{\partial^2}{\partial t^2} + F_3 \frac{\partial^2}{\partial t^2} + \dots)} - 1 \right) \sqrt{P(t)} e^{i\phi(t)} \right]. \quad (20)$$

For studying the transmission of PCM signals, we assume that the chip free modulation can be achieved by using the external modulator. For this assumption, we have $dP/dt = 0$ at the decision point for both 1 and 0 bit. So, ΔP can be expressed as

$$\Delta P = 2Po(2F_2\phi'' - 3F_3\phi''\phi'),$$

$$\frac{\Delta P^2}{Po^2} = 16F_2^2(\phi'')^2 C + 36F_3^2(\phi'')^2(\phi') - 48F_2F_3(\phi'')^2(\phi'). \quad (21)$$

Hence, the intensity of noise level can be calculated as

$$r_i^2 = \frac{32}{3}F_2^2\pi^3\Delta vB^3 + \frac{144}{3}F_3^2\pi^4\Delta v^2B^2 - \frac{192}{3}F_2F_3\pi^4\Delta v^2B^2. \quad (22)$$

The power penalty can be calculated given by Koyama and Suematsu [7,8] as

$$P(r_i) = \frac{Q\sigma t + Q^2q\Delta f}{R(1 - r_i^2Q^2)}, \quad (23)$$

$$\delta pp = -5 \log(10) \left[\frac{P(r_i)}{P(0)} \right], \quad (24)$$

where $P(0)$ is the power sensitivity when $r_i = 0$, R is the responsivity ranges from 0.4 to 0.95, Q is the digital SNR and is defined as the ratio of signal-to-noise current

Insert the value of ' r_i ' into Eq. (23), we have

$$P(r_i) = \frac{Q\sigma t + Q^2q\Delta f}{R \left[1 - \left(\frac{32}{3}F_2^2\pi^3\Delta vB^3 + \frac{144}{3}F_3^2\pi^4\Delta v^2B^2 - \frac{192}{3}F_2F_3\pi^4\Delta v^2B^2 \right) Q^2 \right]}, \quad (25)$$

$$P(0) = \frac{Q\sigma t + Q^2q\Delta f}{R}. \quad (26)$$

By inserting above two equations in Eq. (24), we can compute the power penalty under the effect of higher-order dispersion parameters.

3. Results and discussion

Referring to ITU-T Rec. 653 recommendation [10,11], we assume $\lambda_0 = 1550$ nm, and $\partial^2\tau/\partial\lambda^2 = 0.085$ ps/nm²km. We obtain the dispersion parameters as $F_2 = 12.75 \times 10^{-24}$ L/km, $F_3 = 2.955 \times 10^{-38}$ L/km, $Q = 6$, $B = 10$ GHz, $\Delta v = 10$ MHz to 100 kHz, $r_{F_2F_3} = \frac{32}{3}F_2^2\pi^3\Delta vB^3 + \frac{144}{3}F_3^2\pi^4\Delta v^2B^2 - \frac{192}{3}F_2F_3\pi^4\Delta v^2B^2 = 5.38 \times 10^{-3}$ = intensity noise under the combined effect of F_2 and F_3 $r_{F_2} = \frac{32}{3}F_2^2\pi^3\Delta vB^3 = 5.38 \times 10^{-3}$ is the intensity noise under the effect of F_2 only, $r_{F_3} = \frac{144}{3}F_3^2\pi^4\Delta v^2B^2 = 4.08 \times 10^{-14}$ is the intensity noise under the effect of F_3 only.

We have calculated the amount of Power penalty required for minimize the loss due to higher-order dispersion parameters at different spectral width and optical distance (Fig. 1). From Figs. 1 and 4, it can be seen that the power penalty increases as the optical distance increases but reduces as the spectral width of the light source reduces from 10 MHz to 100 kHz. Also, the rate of power penalty increases in the same amount both for F_2 and the combined effect of $F_2 + F_3$ (Fig. 2). The individual effect of F_3 is a very interesting case when the spectral width reduced to 100 kHz at optical length of 100 km i.e. the power penalty is almost negligible up to optical distance of 70 km, which can be seen from Fig. 3. It is also investigated that at bit rate ($B = 10$ GHz) and spectral line width ($v = 10$ MHz), our

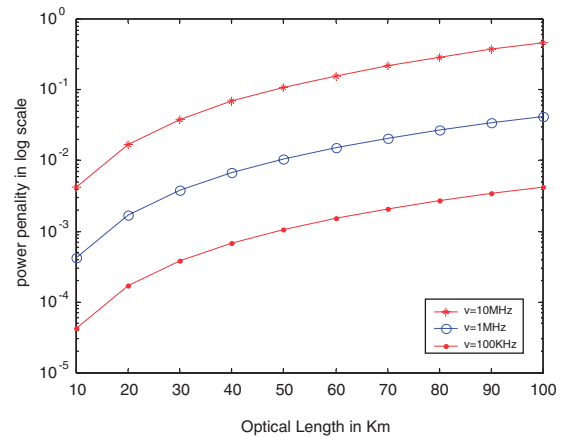


Fig. 1. Power penalty versus optical length up to 100 km at different value of Spectral line width under the effect of F_2 only.

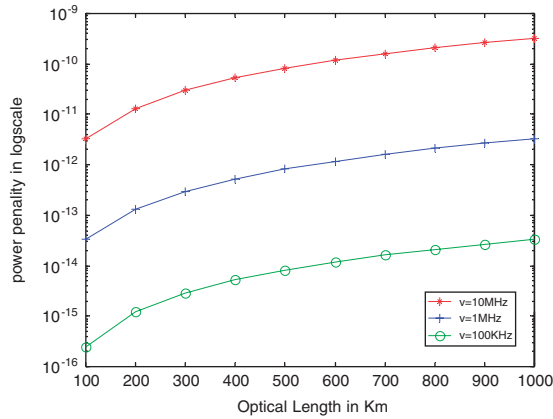


Fig. 2. Power penalty versus optical length up to 1000 km at different value of Spectral line width under the effect of F_3 only.

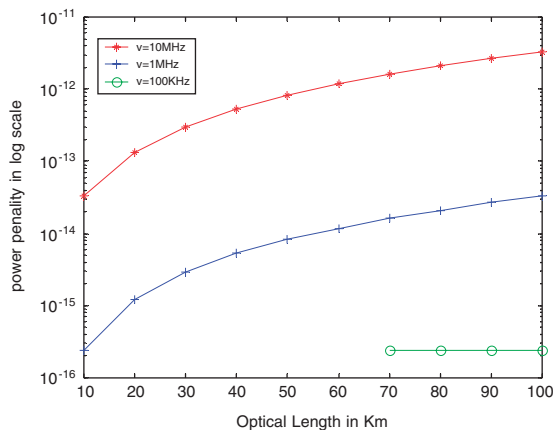


Fig. 3. Power penalty versus optical length up to 100 km at different value of spectral line width under the effect of F_3 only.

values of power penalty is the best match to the values of the practical optical communication systems (Fig. 4).

4. Conclusions

From observations it is concluded that the effect of individual and combined higher-order dispersion parameters have significant impact on the power penalty at different spectral width. It is important to note that the effect of F_3 parameter become significant beyond the transmission length of 70 km. We further investigated that by reducing the spectral width in kHz range, the effect of higher-order dispersion parameters reduces which enhances the transmission distance.

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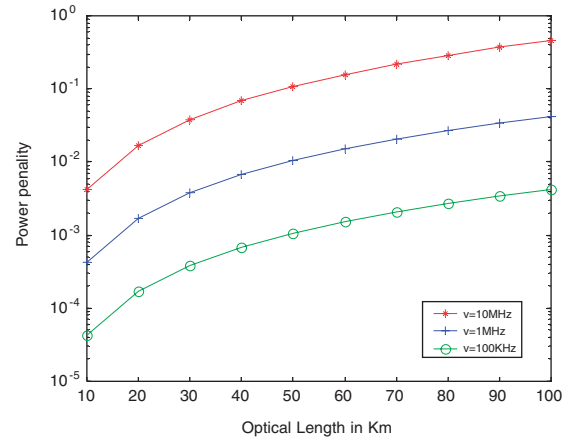


Fig. 4. Power penalty versus optical length up to 100 km at different value of spectral line width under the effect of $F_2 + F_3$.

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