



Original research article

# Improved analysis of four wave mixing with sub-plank higher-order dispersion parameters in optical communication systems

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## ABSTRACT

Four wave mixing (FWM) is one of the major limiting phenomena for wavelength division multiplexed optical communication systems. This paper analysis four wave mixing with sub-plank higher-order dispersion (HOD) parameters up to eighth-order in optical communication through single mode fiber. Four wave mixing power with combination of dispersion parameters up to eighth-order dispersion has been analysed and compared with the dominant second-order dispersion parameter for different channel powers, effective core areas and channel spacing. It has been observed that combined effect of dispersion parameters can bring down four wave mixing by 10–15 dB and facilitate improved inference of four-wave mixing performance with HOD parameters. Hence the results can provide a better direction in the choice of fiber parameters for efficient management of FWM in range of applications.

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## 1. Introduction

Large bandwidth and low maintenance has become mainstay of modern telecommunication industry. Modern Fiber-to-the-home (FTTH) has emerged as one of most practical system to meet requirements of large bandwidth and low maintenance. FTTH require passive optical networks based on Dense Wave Division Multiplexing (DWDM) in which hybrid Raman-Erbium-Doped Fiber Amplifier (EDFA) can also be used due to high bandwidth efficiency [1–3]. In case of DWDM system with multiple optical channels, Four-wave mixing (FWM) and hence nonlinear crosstalk occur as a result of interactions between channels. Crosstalk is strongly dependent on wavelength and is sensitive to input channel power as it increases with increase in channel power [4]. FWM can severely degrade the system performance [5–7]. FWM can be lowered by increasing the effective core area of fiber, using relatively low channel counts, wide channel spacing and utilising fibers with a reasonable degree of dispersion [8,9]. Research is being carried in novel manner to suppress FWM by using advanced modulation formats, advanced dispersion management techniques, use of special fibers as well as low noise amplifiers, forward error correction codes and new system designs like equal channel spacing with alternate channel delay etc. At the

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same time, concept of four wave mixing has been used in parametric amplifiers for signal amplification, phase conjugation and wavelength conversion. FWM has also provided basic technology for reduction of quantum noise as well as generating quantum-correlated photon pairs [10]. Song et al., [11] analysed the FWM in optical fibers in terms of a new phase-matching factor with second-order dispersion parameter.

Recently in advanced communications systems, the need has been felt to go beyond third- order dispersion for wide bandwidth signals such as super-continuum generation. Singh et al., [12] observed the combined effect of higher-order dispersion (HOD) parameters up to fifth-order (5OD), on crosstalk due to FWM. Zhou et. al., [13] obtained exact solutions of the cubic-quintic nonlinear optical transmission equation with HOD terms. In this paper, we have extended work through schrodinger equation [10] by calculating the effect of sub-plank higher-order dispersion parameters up to eighth-order (8OD) on FWM power at different input channel powers, core effective areas and channel spacing. The mathematical and analytical analysis of FWM in presence of sub-plank HOD parameters has enhanced the understanding of FWM phenomena and design of dispersion compensation techniques in DWDM optical systems.

In Section 2, the FWM power in terms of factors like phase matching factor  $\Delta\beta'$ , HOD parameters, channel power, fiber core effective area and channel spacing has been detailed. In Section 3, the numerical results for FWM performance in presence of 2OD and HOD parameter have been discussed and Section 4 summarizes the conclusions.

## 2. Numerical analysis of FWM

For optical transmission through a single mode fiber (SMF) line, the propagation of a signal can be described in the terms of propagation constant ‘ $\beta$ ’. This propagation constant in terms of Taylor series can be expressed as [10]:

$$\beta = \beta_0 + (\omega - \omega_0) \frac{d\beta}{d\omega} + \frac{1}{2}(\omega - \omega_0)^2 \frac{d^2\beta}{d\omega^2} + \frac{1}{6}(\omega - \omega_0)^3 \frac{d^3\beta}{d\omega^3} + \frac{1}{24}(\omega - \omega_0)^4 \frac{d^4\beta}{d\omega^4} + \frac{1}{120}(\omega - \omega_0)^5 \frac{d^5\beta}{d\omega^5} + \frac{1}{720}(\omega - \omega_0)^6 \frac{d^6\beta}{d\omega^6} + \frac{1}{5040}(\omega - \omega_0)^7 \frac{d^7\beta}{d\omega^7} + \frac{1}{40320}(\omega - \omega_0)^8 \frac{d^8\beta}{d\omega^8} + \dots \tag{1}$$

Here,  $\frac{d\beta}{d\omega} = \tau$  is the propagation delay per optical length, Hence Eq. (1) can be written as:

$$\Delta\beta = \beta - \beta_0 = 2\pi \left[ (\Delta f)\tau + \pi(\Delta f)^2 \frac{d\tau}{d\omega} + \frac{2\pi^2}{3}(\Delta f)^3 \frac{d^2\tau}{d\omega^2} + \frac{\pi^3}{3}(\Delta f)^4 \frac{d^3\tau}{d\omega^3} + \frac{2\pi^4}{15}(\Delta f)^5 \frac{d^4\tau}{d\omega^4} + \frac{2\pi^5}{45}(\Delta f)^6 \frac{d^5\tau}{d\omega^5} + \frac{4\pi^6}{315}(\Delta f)^7 \frac{d^6\tau}{d\omega^6} + \frac{\pi^7}{315}(\Delta f)^8 \frac{d^7\tau}{d\omega^7} \dots \right] \tag{2}$$

The dispersion parameters up to 5OD [11,12] are defined as

$$\beta_2 = \frac{d\tau}{d\omega} = \frac{\lambda^2}{2\pi c} \frac{\partial\tau}{\partial\lambda} = \frac{\lambda^2}{2\pi c} D_0 \tag{3}$$

the second-order dispersion (2OD) parameter,

$$\beta_3 = \frac{d^2\tau}{d\omega^2} = \frac{\lambda^2}{(2\pi c)^2} \left[ \lambda^2 \frac{\partial^2\tau}{\partial\lambda^2} + 2\lambda \frac{\partial\tau}{\partial\lambda} \right] = \frac{\lambda^2}{(2\pi c)^2} [\lambda^2 D_1 + 2\lambda D_0] \tag{4}$$

the third-order dispersion (3OD) parameter,

$$\beta_4 = \frac{d^3\tau}{d\omega^3} = \frac{\lambda^3}{(2\pi c)^3} \left[ \lambda^3 \frac{\partial^3\tau}{\partial\lambda^3} + 6\lambda^2 \frac{\partial^2\tau}{\partial\lambda^2} + 6\lambda \frac{\partial\tau}{\partial\lambda} \right] = \frac{\lambda^3}{(2\pi c)^3} [\lambda^3 D_2 + 6\lambda^2 D_1 + 6\lambda D_0] \tag{5}$$

the fourth-order dispersion (4OD) parameter,

$$\beta_5 = \frac{d^4\tau}{d\omega^4} = \frac{\lambda^4}{(2\pi c)^4} \left[ \lambda^4 \frac{\partial^4\tau}{\partial\lambda^4} + 12\lambda^3 \frac{\partial^3\tau}{\partial\lambda^3} + 36\lambda^2 \frac{\partial^2\tau}{\partial\lambda^2} + 24\lambda \frac{\partial\tau}{\partial\lambda} \right] = \frac{\lambda^4}{(2\pi c)^4} [\lambda^4 D_3 + 12\lambda^3 D_2 + 36\lambda^2 D_1 + 24\lambda D_0] \tag{6}$$

the fifth-order dispersion (5OD) parameter. Dispersion parameters up to  $\beta_8$  (i.e. 8OD) have been further calculated as

$$\beta_6 = \frac{d^5\tau}{d\omega^5} = \frac{\lambda^5}{(2\pi c)^5} [\lambda^5 D_4 + 20\lambda^4 D_3 + 120\lambda^3 D_2 + 240\lambda^2 D_1 + 120\lambda D_0] \tag{7}$$

the sixth-order dispersion (6OD) parameter,

$$\beta_7 = \frac{d^6 \tau}{d\omega^6} = \frac{\lambda^6}{(2\pi c)^6} [\lambda^6 D_5 + 30\lambda^5 D_4 + 300\lambda^4 D_3 + 1200\lambda^3 D_2 + 1800\lambda^2 D_1 + 720\lambda D_0] \quad (8)$$

the seventh-order dispersion (7OD) parameter and

$$\beta_8 = \frac{d^7 \tau}{d\omega^7} = \frac{\lambda^7}{(2\pi c)^7} [\lambda^7 D_6 + 42\lambda^6 D_5 + 630\lambda^5 D_4 + 4200\lambda^4 D_3 + 12600\lambda^3 D_2 + 15120\lambda^2 D_1 + 5040\lambda D_0] \quad (9)$$

the eighth-order dispersion (8OD) parameter.

Where  $\frac{\partial \tau}{\partial \lambda} = D_0$ ,  $\frac{\partial^2 \tau}{\partial \lambda^2} = D_1$ ,  $\frac{\partial^3 \tau}{\partial \lambda^3} = D_2$ ,  $\frac{\partial^4 \tau}{\partial \lambda^4} = D_3$ ,  $\frac{\partial^5 \tau}{\partial \lambda^5} = D_4$ ,  $\frac{\partial^6 \tau}{\partial \lambda^6} = D_5$ ,  $\frac{\partial^7 \tau}{\partial \lambda^7} = D_6$

The first term in Eq. (2) is neglected as it has no influence on distortion of signal and replacing the values of higher-order dispersion parameters  $\beta_2$  to  $\beta_8$  from the Eqs. (3)–(9) in Eq. (2), we get

$$\begin{aligned} & D_0 + \frac{\Delta f \lambda}{3c} (\lambda D_1 + 2D_0) + \frac{\Delta f^2 \lambda^2}{12c^2} (\lambda^2 D_2 + 6\lambda D_1 + 6D_0) \\ & + \frac{\Delta f^3 \lambda^3}{60c^3} (\lambda^3 D_3 + 12\lambda^2 D_2 + 36\lambda D_1 + 24D_0) \\ \Delta \beta = \frac{\pi \Delta f^2 \lambda^2}{c} & \left[ \frac{\Delta f^4 \lambda^4}{360c^4} (\lambda^4 D_4 + 20\lambda^3 D_3 + 120\lambda^2 D_2 + 240\lambda D_1 + 120D_0) + \right. \\ & \frac{\Delta f^5 \lambda^5}{2520c^5} (\lambda^5 D_5 + 30\lambda^4 D_4 + 300\lambda^3 D_3 + 1200\lambda^2 D_2 + 1800\lambda D_1 + 720D_0) + \\ & \left. \frac{\Delta f^6 \lambda^6}{20160c^6} (\lambda^6 D_6 + 42\lambda^5 D_5 + 630\lambda^4 D_4 + 4200\lambda^3 D_3 + 2600\lambda^2 D_2 + 15120\lambda D_1 + 5040D_0) \right. \\ & \left. + \dots \right] \quad (10) \end{aligned}$$

Where,  $\Delta f$  is channel spacing,  $D_0$  is fiber chromatic dispersion,  $\alpha$  is attenuation factor and  $c$  is velocity of light.

The FWM power in WDM systems [11,12], [14] is given as:

$$P_{FWM} = \frac{\eta'}{9} D^2 \gamma^2 P_1 P_2 P_3 \exp(-\alpha L) \left[ \frac{1 - \exp(-\alpha L)^2}{\alpha^2} \right] \quad (11)$$

and the FWM conversion efficiency is expressed as:

$$\eta' = \frac{\alpha^2}{\alpha^2 + (\Delta \beta')^2} \left[ 1 + \frac{4 \exp(-\alpha L) \sin^2(\Delta \beta' L/2)}{(1 - \exp(-\alpha L))^2} \right] \quad (12)$$

The Intensity dependent Phase matching factor  $\Delta \beta'$  [15] can be defined as:

$$\Delta \beta' = \Delta \beta - \gamma m (P_1 + P_2 - P_3) \left[ \frac{\{1 - \exp(-\alpha L_{eff})\}}{\alpha L_{eff}} \right] \quad (13)$$

where  $m$  is integer. For long fiber ( $L \gg L_{eff}$ ) the effective fiber length is defined as:

$$L_{eff} = \frac{1 - \exp(-\alpha L)}{\alpha} \approx \frac{1}{\alpha}$$

Here  $\gamma$  = Non-linear coefficient =  $\frac{2\pi n_2}{\lambda A_{eff}}$

$n_2 = 2.68 \times 10^{-20} m^2/W$  is optical fiber non-linear refractive index,

$P_1, P_2, P_3$  are input channel powers,  $A_{eff}$  is effective area of fiber core and  $L_{eff}$  is effective fiber length.

As per ITU recommendation G.653 the input channel power of the three input channels has been assumed to be equal such that  $P_1 = P_2 = P_3 = P_0$ . The optical system is numerically analysed in the MATLAB environment.

### 3. Results and discussion

For the standard single mode fiber (SSMF), with combinations of dispersion parameters up to eighth-order, FWM power is analysed for various channel powers ranging from 10 mW to 100 mW, with the following dispersion parameters:

$D_0$  = Fiber chromatic dispersion = 17 ps/km-nm,  $\frac{\partial D}{\partial \lambda}$  = Dispersion slope = 0.09 ps/km-nm<sup>2</sup>

$\frac{\partial^2 D}{\partial \lambda^2}$  = Dispersion curvature = 0.00025 ps/km-nm<sup>3</sup>

$\frac{\partial^3 D}{\partial \lambda^3}$  = Dispersion curvature = 0.0000025 ps/Km-nm<sup>4</sup>,  $D_4 = 0.000000025$  ps/km-nm<sup>5</sup>

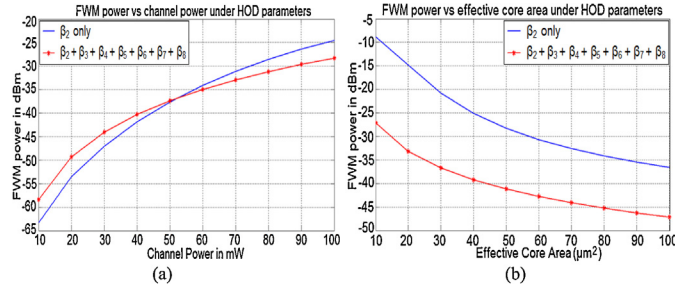


Fig. 1. (a) FWM power versus input channel power and (b) FWM power versus effective core area.

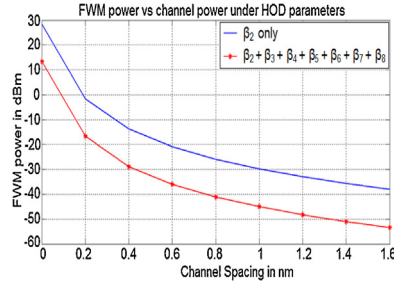


Fig. 2. FWM power and channel spacing.

$D5 = 0.0000000025 \text{ ps/km-nm}^6$ ,  $\alpha = \text{Attenuation factor} = 0.35 \text{ dB/km}$

Value of  $D6$  is further considered to be very small. The pump wavelengths have been taken as 1558 nm and 1558.8 nm such that channel spacing of 0.8 nm is obtained.

Following the numerical results, the FWM power for combination of HOD from  $\beta_2$  to  $\beta_8$  (i.e.  $\sum_{n=2}^8 n\text{OD}$ ) is analysed and compared with the dominant second-order (2OD) dispersion parameter ( $\beta_2$ ) for a fiber core effective area of  $55 \mu\text{m}^2$  and at a fiber length of 2 km, for different values of channel powers. It is noticed [Fig. 1(a)] that with the increase in channel power, FWM increases because the nonlinear effects depend on ratio of light power to cross sectional area of the fiber. Beyond 50 mW channel power, combination of HOD parameters suppresses the FWM power. FWM power is noted to vary from (–63 to –25 dBm) and (–58 to –28 dBm) for 2OD and  $\sum_{n=2}^8 n\text{OD}$  respectively.

The variation in FWM power for different fiber effective core area is also analysed with 2OD and  $\sum_{n=2}^8 n\text{OD}$  combination at 40 mW channel power [Fig. 1(b)]. It is noticed that FWM decreases as the core effective area is increased from  $10 \mu\text{m}^2$  to  $100 \mu\text{m}^2$ . This is because, with increase in effective core area, the intensity in the fiber decreases, thus the nonlinearities also decrease as effect of nonlinearity is directly proportional to intensity. For a given channel power as the core effective area is increased from  $10 \mu\text{m}^2$  to  $100 \mu\text{m}^2$ , FWM for  $\sum_{n=2}^8 n\text{OD}$  combination gets improved by 10 dB. It is observed that for any particular effective core area the FWM power is lower by up to 10 dB for  $\sum_{n=2}^8 n\text{OD}$  in contrast to individual 2OD parameter. FWM power is observed (–9 to –36 dBm) and (–26 to –47 dBm) for 2OD and  $\sum_{n=2}^8 n\text{OD}$  respectively. Hence in optical communication systems, a fiber of bigger effective core area can be selected for reduced level of FWM, while a fiber of smaller effective core area can be chosen to get wavelength conversion.

Further, the effect on FWM is analysed for different channel spacing ranging from 0 to 1.6 nm with 2OD and  $\sum_{n=2}^8 n\text{OD}$  combination at a given channel power (Fig. 2). It is noticed that with increase in channel spacing, FWM decreases because of lesser interference between input frequencies and for any particular channel spacing the FWM power is lower by up to 15 dB for  $\sum_{n=2}^8 n\text{OD}$  combination in contrast to that of individual 2OD parameter. FWM power is measured as (28 to –38dBm) and (13 to –53dBm) for 2OD and  $\sum_{n=2}^8 n\text{OD}$  respectively. Hence an improved analysis of FWM with sub-plank HOD parameters in optical communication systems is obtained.

#### 4. Conclusion

In this paper we have presented the detailed numerical analysis of influence of higher-order dispersion parameters on FWM. It has been demonstrated analytically and mathematically that presence of higher-order dispersion parameters bring down the FWM power by 10–15 dB. After 50 mW channel power, combination of dispersion parameters suppresses the FWM power. Hence for higher pump powers the combinations of dispersion parameters result in improved performance of optical communication systems. Our results can facilitate optimum choice of fiber parameters in modern fiber optics systems and can be utilized for designing suitable dispersion compensation techniques for long distance communication systems.

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