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Asymptotic multicast throughput analysis and energy efficiency in WSN under double Nakagami fading channel using Extreme value theory

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ABSTRACT

Wireless sensor network (WSN) applications demand high throughput and energy efficiency for commercial adaptability. Although recent research enhances our knowledge about traditional fading, yet the shortcomings like mathematical complexity, absence of closed solution, and lack of physical insight of random behavior impede analytical study. Extreme value theory (EVT) has been used in cases where the direct solution is almost intractable. This prompts us to use EVT, in relatively more complex fading environments like double Nakagami. Here, we present an asymptotic analysis of multicasting throughput and energy efficiency for WSN communications under the assumption of independent and identically distributed channels. Considering the cumulative distribution function (CDF) of the SNR, we obtain expressions for minimum scaled SNR. The distribution of minima extreme is shown to converge to Weibull distribution and subsequently the effects of minimum scaled SNR on throughput are obtained. The influence of the varying number of users, fading strength and SNR on the per user throughput are also explored. For validation, we compare our results with recently reported approximately exact approach and find a satisfactory agreement. We also perform the Monte-Carlo simulations. This study may help in designing throughput friendly and energy-efficient systems.

1. Introduction

WSN applications are showing their presence in almost all aspects of life, from simple health care monitoring to challenging remote volcanic eruption alarming systems. A well designed WSN can provide enhanced channel capacity and improved energy-efficiency for the supported applications. WSN applications are found in many fields like wildlife animal tracking and military warfare [1].

The received signal in a wireless communication system suffers from some damaging effects across channel. Radio propagations are subject to effects of fading, shadowing and path-loss [2]. Impairment path-loss depends on the distance, while fading and shadowing impairments are studied by statistical models. For improved performance and enhanced reliability of the underlying communications, the significance of research in accurate channel modeling increases and it becomes important to model the system behavior characterized by the combined effect of both fading and shadowing, as they seldom take place in isolation [3,4].

For different fading and/or shadowing scenarios, a large number of statistical models have been put forward e.g., popular Rayleigh, Rice [5], Nakagami-*m* [6], Weibull [7], log-normal distribution [8,9], double Rayleigh fading [10], double Nakagami-*m* fading [11]. Here, we have considered communications operating under double Nakagami fading channels, and used Extreme value theory to analyze the per user throughput of a system using multicasting approach for later's suitability in effectively disseminating data to multiple receivers as compared to unicast approach. High communication reliability and throughput is desired for multicast applications to be commercially adaptable and deliver benchmark performance even under the presence of impairments. A number of examples illustrating multicast applications adapted in WSN are available in [12,13].

Because of strengths of EVT, the performance of a given system can be better analyzed by studying the asymptotic distribution of extremes. Some recent research efforts include, derivation of rationale expressions for average throughput of system using homogeneous as well as heterogeneous signal-to-noise ratio (SNR)

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in [14]. The throughput levels in the latter case were less than for the former case. In [15], the authors used EVT concepts to asymptotically investigate throughput of multicast and unicast communications by taking into consideration the joint impact of user distribution (random) and Rayleigh fading. The authors in [16] investigated the capacity of double Nakagami fading channels by deriving expressions and studied the impact of amount of fading on the derived attributes related to capacity of the channel. The authors in [17] derived the asymptotic bound and observed that when the outage probability constraint is small, the asymptotic expression can be seen the reasonable approximation to the exact expression, and the accuracy of asymptotic bound is tighter than the upper and lower bounds. The authors in [18] using Extreme value theory computed different moments of the maxima and related order statistics of interest. Expressions for spectral efficiency and scheduling gains were derived for the Rayleigh and Composite fading models using Extreme value theory in [19].

In context to energy aspects, some of research contributions include setting of physical layer parameters to enhance the energy efficiency of WSN by deriving an expression for the energy consumption in [20], evaluation of various energy efficient techniques for multicast communications in [21] and investigation of the energy utilization for WSN based on BPSK communications over the Generalized-*K* shadowed fading channel considering the impact of packet retransmissions and repeated training sequences in [22].

Although quite a good amount of work has been done in simple fading environments like Rayleigh fading [15], yet we are posed with a challenge to mathematically model the complex fading scenario, leading our research to use Extreme value theory [23], in relatively more complex and relatively lesser explored fading environments like double Nakagami fading. Although, recent research efforts have focused on performance modeling of communication operating under the effects of complex fadings, but asymptotic derivation of multicasting throughput of complex fading environments like double-Nakagami fading has not been widely investigated. These have their applications in the vehicle–vehicle communication applications and the keyhole channels [24]. Therefore, our research focus considers the performance evaluation of communications operating over double Nakagami channel in terms of the asymptotic multicasting throughput using Extreme value theory. EVT is being used because this approach being more concise and it is easier to get insights and can be applied to wider range of channel fading models [14]. Some of the useful insights related to EVT are given in Appendices A–C.

In the present work, we considered cumulative distribution function of SNR of double Nakagami fading to obtain asymptotic expressions for minimum scaled SNR. The distribution of minima extreme was shown to converge to Weibull distribution and then subsequently the effects of minimum scaled SNR on the multicasting throughput were obtained. We finally obtained the results of multicasting throughput by varying the number of users, severity of fading and SNR. We further obtained the results related to energy efficiency. For the sake of completeness and validation, we compared our results with results from the recently reported approximately exact analysis approach [25], wherein tight approximation as in [26] has been used to obtain an approximately exact effective rate for different fading environments. We also performed Monte-Carlo simulations.

The rest of paper is arranged as follows. In Section 2, we perform the throughput and energy efficiency analysis by using asymptotic approximation analysis and approximately exact analysis approach. Results are reported, compared and discussed in Section 3. The conclusions of the study are given in Section 4.

2. System model

In the system under consideration, a node is assumed to communicate same data to a group of nodes by employing omni-directional antenna, to facilitate hearing for the nodes in the transmission coverage. The multicast throughput of this arrangement is determined by the performance of the least throughput-friendly user among the pool at any given time. We consider the communications with an assumption of independent and identically distributed channels operating over double Nakagami fading, where the channel is modeled as two cluster scenario, with each cluster having identical Nakagami-*m* fading environment.

Wireless communications are prone to the loss of power mainly due to fading (short-term fading) and multiple scattering (shadowing) and realistically, none of these occur in isolation, rather fading occurs along with shadowing so the effects of both these random entities must be jointly considered in performance investigations of WSNs [2]. This is based on the fundamental idea of clustering, where various scattering centers are organized into different clusters so that signals may reach the receiver after multiple scattering among them, instead of arriving independently from each scattering center. For multiple scattering, we consider at least two such clusters during the communication. The received power (P_R) at receiver is equal to product of powers of two clusters i.e. $P_R = P1 \cdot P2$, where both $P1$ and $P2$ are gamma distributed. If both the clusters are identical in Nakagami-*m* PDF for the envelope, the PDF of received power and that of SNR can be conveniently obtained. This model organization for shadowed channel is often called Double Nakagami model [27, 4.6]. Furthermore, every individual cluster consists of mini-clusters and the scattering within that results in fading. Using CDF of SNR of double Nakagami fading, we asymptotically derive expressions for minimum scaled SNR. The minima extreme distribution is shown to belong to the domain of attraction of the Weibull distribution and then subsequently the effects of minimum scaled SNR on the multicasting throughput are investigated. Finally, considering the total throughput and total power consumption of the underlying circuitry we evaluate another critical performance metric Network energy efficiency.

In the subsections to follow, first, we discuss the asymptotic derivation of an expression for minimum scaled SNR and multicasting throughput rate. Thereafter, we discuss the energy efficiency aspects and then we present the approximately exact analysis approach used to validate our results.

2.1. Asymptotic approximation approach

Here, we use Extreme value theory to obtain an expression for minimum scaled SNR distribution and then subsequently derive the expression for multicast rate distribution for the double Nakagami fading environment, following a procedure similar to that used for other fading models in [14,15].

2.1.1. Deriving minimum scaled SNR distribution using EVT

In the multicast communications, the minimum multicast rate corresponds to the minimum scaled SNR, therefore, we need to find the distribution of the minimum rate of all the users. We apply EVT to find asymptotic approximation of the CDF of the SNR of double Nakagami channel which is obtained from the CDF of composite fading model as given in [27, Eq. 4.298] with $c = m$, where m and c are fading severity parameters.

$$F_Z(z) = \frac{2}{\Gamma(m)\Gamma(m+1)}\left(\frac{m^2 z}{Z_0}\right)^m \times {}_1F_2\left([m], [1+m, 1], \frac{m^2 z}{Z_0}\right) \tag{1}$$

where z is the SNR, Z_0 is the average SNR, $\Gamma(\cdot)$ is the gamma function and pFq refers to the hypergeometric function. Using the definition of Hypergeometric function [28, Section 1], $F(\cdot)$ can be written as

$$ {}_1F_2(\alpha;\beta,\gamma;x)=\sum_{r=0}^{\infty}\frac{(\alpha)_r}{(\beta)_r(\gamma)_r}\frac{x^r}{r!} \tag{2}$$

where $(\alpha)_r$ is the Pochhammer symbol [29, 1.0.1]

$(\alpha)_r=\alpha(\alpha+1)\ldots(\alpha+r-1),\quad (\alpha)_0=1,\quad \alpha\neq 0$

$$\therefore\quad {}_1F_2\left([m],[1+m,1],\frac{m^2z}{Z_0}\right)=\sum_{i=0}^{\infty}\frac{(m)_i}{(1+m)_i\ 1_i}\left(\frac{m^2z}{Z_0}\right)^i\times\frac{1}{i!} \tag{3}$$

Using following results

$$\frac{m_i}{(1+m)_i}=\frac{m}{(m+i)}\quad\text{and}\quad 1_i=i!$$

$$\text{we get,}\quad {}_1F_2\left([m],[1+m,1],\frac{m^2z}{Z_0}\right)=\sum_{i=0}^{\infty}\frac{m}{(m+i)}\left(\frac{m^2z}{Z_0}\right)^i\times\frac{1}{(i!)^2} \tag{4}$$

Using Eq. (4), Eq. (1) reduces to

$$F_Z(z)=\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2z}{Z_0}\right)^{m+i}\times\frac{1}{(i!)^2} \tag{5}$$

Now we apply EVT [23] to find the minima of distribution and corresponding asymptotic expression for minimum scaled SNR. Using Eq. (C.1) given in Appendix C

$\alpha(F_Z)=Inf\quad\{z,F_Z(z)>0\}$

we get,

$\alpha(F)=0$

Further, using Eq. (C.2) given in Appendix C, if $\alpha(F)>-\infty$ and the function $F_Z^*(z)=F_Z\left(\alpha(F)-\frac{1}{z}\right);\quad z<0$ then we can apply Eq. (C.3) given in Appendix C to obtain the following expression

$$\lim_{t\to-\infty}\frac{F_Z^*(tz)}{F_Z^*(t)}=\lim_{t\to-\infty}\frac{\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2}{Z_0\ tz}\right)^{m+i}\times\frac{1}{(i!)^2}}{\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2}{Z_0\ t}\right)^{m+i}\times\frac{1}{(i!)^2}} \tag{6}$$

To have an idea of details, we expand the summation

$$\lim_{t\to-\infty}\frac{F_Z^*(tz)}{F_Z^*(t)}=\lim_{t\to-\infty}\frac{\frac{m}{m}\left(\frac{m^2}{Z_0\ tz}\right)^{m}+\left(\frac{m}{m+1}\left(\frac{m^2}{Z_0\ tz}\right)^{m+1}\times\frac{1}{1^2}\right)+\left(\frac{m}{m+2}\left(\frac{m^2}{Z_0\ tz}\right)^{m+2}\times\frac{1}{2^2}\right)+\cdots}{\frac{m}{m}\left(\frac{m^2}{Z_0\ t}\right)^{m}+\left(\frac{m}{m+1}\left(\frac{m^2}{Z_0\ t}\right)^{m+1}\times\frac{1}{1^2}\right)+\left(\frac{m}{m+2}\left(\frac{m^2}{Z_0\ t}\right)^{m+2}\times\frac{1}{2^2}\right)+\cdots} \tag{7}$$

Differentiating the numerator and the denominator with respect to t and applying L'Hospital rule once, in Eq. (7), we get

$$\lim_{t\to-\infty}\frac{F_Z^*(tz)}{F_Z^*(t)}=z^{-m} \tag{8}$$

Since, $m>0$, and γ of Eq. (C.3) given in Appendix C is m, therefore $\gamma(=m)>0$. Now, as per the theorem given in Appendix C, if $\gamma>0$, then there exist constants c_N and d_N such that $(Z_{min,N}-c_N)/d_N$ has convergence of Weibull distribution for minima, where $c_N=\alpha(F)$ and $d_N=F_Z^{-1}\left(\frac{1}{N}\right)-\alpha(F)$.

$$\text{Since,}\quad \alpha(F)=0,\quad\therefore\quad c_N=0 \tag{9}$$

$$d_N=F_Z^{-1}\left(\frac{1}{N}\right)-\alpha(F)=F_Z^{-1}\left(\frac{1}{N}\right)=z_{\frac{1}{N}} \tag{10}$$

Now, it is cumbersome to find value for d_N, therefore we proceed to find it approximately. When N is large, $\frac{1}{N}$ is small and, therefore correspondingly $z_{\frac{1}{N}}$ is small.

$$\therefore,\quad \frac{1}{N}=\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2\ z_{\frac{1}{N}}}{Z_0}\right)^{m+i}\times\frac{1}{(i!)^2} \tag{11}$$

The series on the right hand side satisfies D'Alembert's Criterion for convergence for $\left(\frac{m^2\ z_{\frac{1}{N}}}{Z_0}\right)\ll 1$, Therefore, we find approximate value as follows by considering dominating term of the series. Therefore, from Eq. (11), we get

$$\left(z_{\frac{1}{N}}\right)^m=\frac{\Gamma(m)\Gamma(m+1)}{2N}\left(\frac{Z_0}{m^2}\right)^m \tag{12}$$

$$z_{\frac{1}{N}}=\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}\left(\frac{Z_0}{m^2}\right) \tag{13}$$

$$\therefore,\quad d_N=\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}\left(\frac{Z_0}{m^2}\right) \tag{14}$$

Using value of d_N given in Eq. (14), the limiting distribution for the minimum scaled SNR can be found by using following expression

$$\lim_{N\to\infty}F_{Z_{\min},N}(z,d_N)\simeq 1-e^{-z/d_N} \tag{15}$$

Further, using expressions derived above, we find the multicasting rate for the communication operating under the effects of double Nakagami fading in the next subsection.

2.1.2. Finding multicast rate distribution using EVT

The multicast rate which is expressed as $R_i(Z_i)=B\times\log_2(1+Z_i)$ is a monotonically increasing function of Z_i. Since, minimum throughput rate is a random variable as it is obtained from another random variable $Z_{min,N}$, therefore, in the following we find its distribution of the minima. Minimum throughput $R_{min,N}$ is expressed as

$$R_{\min,N}=B\times\log_2(1+Z_{\min,N}) \tag{16}$$

The CDF of multicasting rate is expressed as

$$F_R(r)=F_Z(2^{r/B}-1) \tag{17}$$

To find the distribution of minima, we apply Theorem given in Appendix C again using Eq. (C.1) and (C.3).

$\alpha(F_Z)=Inf\{r:F_Z(r)>0\}\quad\therefore,\quad\alpha(F)=0$

As $\alpha(F)>-\infty$ and the function $F_Z^*(r)=F_Z\left(\alpha(F)-\frac{1}{r}\right);\quad r<0$. Further, substituting, $\left(\frac{1}{tz}\right)$ by $\left(2^{\frac{1}{trB}}-1\right)$ and $\left(\frac{1}{t}\right)$ by $\left(2^{\frac{1}{tB}}-1\right)$ on the right hand side of the Eq. (6), we get

$$\lim_{t\to-\infty}\frac{F_Z^*(tr)}{F_Z^*(t)}=\lim_{t\to-\infty}\frac{F_Z^*\left(2^{\frac{1}{trB}}-1\right)}{F_Z^*\left(2^{\frac{1}{tB}}-1\right)} \tag{18}$$

$$\lim_{t\to-\infty}\frac{F_Z^*(tr)}{F_Z^*(t)}=\lim_{t\to-\infty}\frac{\left[\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2}{Z_0}\left(2^{\frac{1}{trB}}-1\right)\right)^{m+i}\times\frac{1}{(i!)^2}\right]}{\left[\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2}{Z_0}\left(2^{\frac{1}{tB}}-1\right)\right)^{m+i}\times\frac{1}{(i!)^2}\right]} \tag{19}$$

Now, to apply L'Hospital rule, we expand the factors in Eq. (19), and differentiate numerator and denominator with respect to t, and get

$$\lim_{t\to-\infty}\frac{F_Z^*(tr)}{F_Z^*(t)}=\lim_{t\to-\infty}\frac{\frac{1}{rB}ln2\cdot\frac{1}{t^2}\cdot\frac{1}{2^{trB}}\left[\left(\frac{m^2}{Z_0}m\left(2^{\frac{1}{trB}}-1\right)^{m-1}\right)+\left(\frac{m}{m+1}\frac{m^2}{Z_0}^{m+1}(m+1)\cdot\left(2^{\frac{1}{trB}}-1\right)^{m}\right)+\cdots\right]}{\frac{1}{B}ln2\cdot\frac{1}{t^2}\cdot\frac{1}{2^{tB}}\left[\left(\frac{m^2}{Z_0}m\left(2^{\frac{1}{tB}}-1\right)^{m-1}\right)+\left(\frac{m}{m+1}\frac{m^2}{Z_0}^{m+1}(m+1)\cdot\left(2^{\frac{1}{tB}}-1\right)^{m}\right)+\cdots\right]} \quad (20)$$

Taking limit t as $-\infty$, the term $\left(2^{\frac{1}{trB}}-1\right)$ becomes 0, therefore Eq. (20) reduces to $\frac{1}{r}=r^{-1}$ which corresponds to $\gamma=1$ as in Eq. (C.3) given in Appendix C. Therefore, random value for multicast rate expressed as $R_{min,N}=B\log_2(1+Z_{min,N})$ will converge to the Weibull distribution (i.e. exponential with $\gamma=1$). Therefore, we express the normalizing constants as follows

$$c_N=\alpha(F)=0$$

$$d_N=B\log_2\left[1+F_Z^{-1}\left(\frac{1}{N}\right)-\alpha(F)\right] \quad (21)$$

$$d_N=B\log_2\left[1+F_Z^{-1}\left(\frac{1}{N}\right)\right]$$

Assuming N is large, $\frac{1}{N}$ is small and correspondingly $z_{\frac{1}{N}}$ is also very small. Therefore, $d_N=F_Z^{-1}\left(\frac{1}{N}\right)=z_{\frac{1}{N}}$.

$$\therefore,\quad \frac{1}{N}=\frac{2}{\Gamma(m)\Gamma(m+1)}\sum_{i=0}^{\infty}\frac{m}{m+i}\left(\frac{m^2z_{\frac{1}{N}}}{Z_0}\right)^{m+i}\times\frac{1}{(i!)^2} \quad (22)$$

Now, since finding the exact value of d_N is difficult, therefore we can find an approximate value of d_N by taking first term of the convergent series $\left(\frac{m^2z_{\frac{1}{N}}}{Z_0}\ll 1\right)$ in Eq. (22)

$$\frac{1}{N}=\frac{2}{\Gamma(m)\Gamma(m+1)}\cdot\left(\frac{m^2z_{\frac{1}{N}}}{Z_0}\right)^m$$

$$\left(\frac{m^2z_{\frac{1}{N}}}{Z_0}\right)^m=\frac{\Gamma(m)\Gamma(m+1)}{2N}$$

$$\left(\frac{m^2z_{\frac{1}{N}}}{Z_0}\right)=\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}$$

$$d_N=z_{\frac{1}{N}}\simeq\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}\cdot\frac{Z_0}{m^2} \quad (23)$$

The multicast rate per user is approximated as given below (using Eq. (23))

$$R_M\simeq B\log_2\left(1+\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}\cdot\frac{Z_0}{m^2}\right) \quad (24)$$

Total multicast throughput ($R_{M,total}$) can be obtained as $N\times R_M$, because this rate reaches to all users simultaneously.

$$R_{M,total}=N\times R_M \quad (25)$$

$$=N\times B\log_2\left(1+\left(\frac{\Gamma(m)\Gamma(m+1)}{2N}\right)^{\frac{1}{m}}\cdot\frac{Z_0}{m^2}\right) \quad (26)$$

2.2. Energy efficiency

Since energy is a scarce resource in WSNs, therefore network energy efficiency (η_{EE}) is another critical WSN performance metric. It is defined as the ratio of total throughput of the network to the total power consumption.

$$\eta_{EE}=\frac{Throughput}{Power\ Consumption} \quad (27)$$

The total throughput of network has been asymptotically derived using EVT in Section 2.1 and now we further the compute power consumption for the multicasting system under consideration. The total power consumption can be split into power consumption of the amplifiers (P_{PA}) and power consumption of all other circuit blocks (P_{CKT}). The first component P_{PA} is approximated as

$$P_{PA}=(1+\eta)P_T \quad (28)$$

where P_T – transmit power, $\eta=\frac{\xi}{\zeta}-1$ with ζ – amplifier drain efficiency and ξ – peak-to-average power ratio (PAR) [30]. Considering squared power path-loss, P_T is computed as follows [31, Eq. 5.5.6]

$$P_T=\bar{E_b}\frac{R_b(4\Pi d)^2}{G_tG_r\lambda^2}\times L_{margin}\times N_{figure} \quad (29)$$

where ($\bar{E_b}$) – average energy required per bit against acceptable error levels, R_b – data rate in bits/sec, d – transmission distance, G_t and G_r – gains of transmitter and receiver antenna, λ – carrier wavelength, L_{margin} – link margin and N_{figure} – noise figure of receiver defined as $N_{figure}=\frac{N_r}{N_o}$ with N_o = –171 dBm/Hz representing one-sided thermal noise Power Spectral Density (PSD) and N_r – PSD of the total effective noise at the input of the receiver.

Further, evaluation of P_{CKT} include power consumption of circuit blocks like mixers (P_{MIX}), filters (P_{FLT}), digital-to-analog (DAC) component (P_{DAC}) and frequency synthesizers (P_{SYN}). Therefore, it is expressed as sum of powers consumed by circuit blocks excluding the power amplifier at the transmitter side and all blocks at the receiver side.

Now, total power consumption in case of multicasting environment is approximated as sum of power consumed in transmission from one node and sum of all the powers consumed in the transmission paths and that at the receiving nodes.

2.3. Approximately exact analysis approach

For validation of results obtained by asymptotic approximation presented above, we follow the approximately exact analysis approach for multicasting throughput computation as in [25], where authors derived the expression for the exact effective rate for different fading environments using tight approximation as in [26]. Effective throughput for Generalized-K fading model with N users is (**T**), which can be easily applied to double Nakagami fading scenario with small parameter adjustments.

$$\mathbf{T}\approx-\frac{1}{A}\log_2\left({}_3F_0\left(A,\grave{c},\grave{m};-;-\frac{\grave{\Omega}\rho}{\grave{c}\ \grave{m}\ \ N}\right)\right) \quad (30)$$

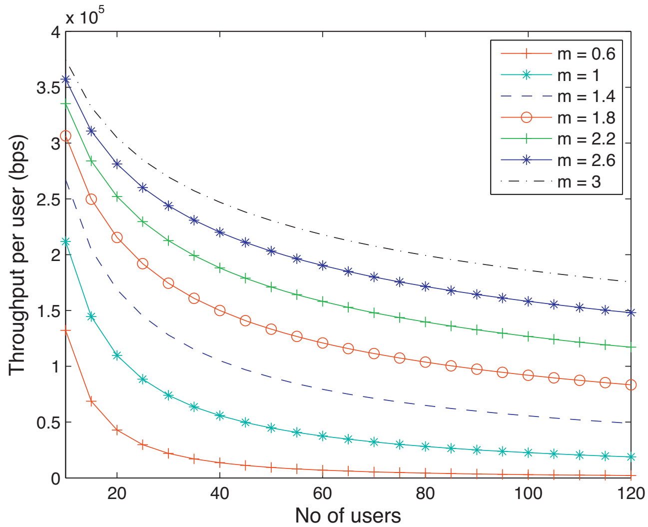

Fig. 1. Multicast throughput per user versus number of users by varying severity of fading parameter for communications operating under double Nakagami fading channel.

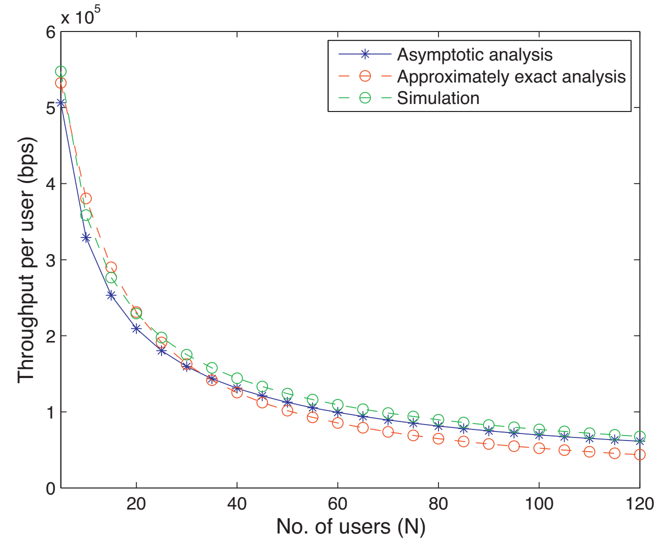

Fig. 2. Multicast throughput per user versus number of users for communications operating under double Nakagami fading channel.

with ‘m, ‘Ω, and ‘c expressed as

$$\grave{m} = mN \tag{31}$$

$$\grave{\Omega} = \Omega N \tag{32}$$

$$\grave{c} = cN + (N-1)\frac{-0.127 - 0.95c - 0.0058m}{1 + 0.00124c + 0.98m} \tag{33}$$

where c and m are fading and shadowing severity fading parameters, Ω is the average fading power, ρ is the average SNR and A is a constant dependent on the bandwidth, block length and QoS requirements.

3. Results and discussion

Here, we undertake the discussion of the results related to (i) multicasting throughput per user by varying number of users and severity of fading, (ii) multicasting throughput per user by varying SNR, (iii) energy efficiency. We also performed validation of the results obtained by asymptotic approximation with those obtained from approximately exact analysis approach and Monte-Carlo simulations.

3.1. *Effect of number of users and severity of fading on multicasting throughput per user*

Fig. 1 shows the variation of multicasting throughput obtained by asymptotic analysis along with number of users for different levels of severity of fading for communications operating over double Nakagami fading. It is observed that the multicasting throughput decreases both with increase in number of users and with increase in severity of fading i.e with decrease in values of parameter m.

Further, to validate our results, the results of the present work with EVT are compared with the results of approximately exact analysis and Monte-Carlo simulation for a specific value of severity parameter m.

Fig. 2 shows the results of multicasting throughput per user obtained from asymptotic approximation plotted along with the throughput obtained from the approximately exact analysis approach and Monte-Carlo simulation approach for the varying number of users operating over double Nakagami fading. It is observed that for the WSN system under consideration, the multicasting throughput per user decreases rapidly with increase in the number of users when the number of users is small, after that variation is slow with further increase in number of users. The results demonstrate a satisfactory match between our results of asymptotic approximation and those of approximately exact analysis approach and simulation approach. Furthermore, the results of asymptotic expression remain accurate, even when number of users increases.

3.2. *Effect of varying SNR on multicasting throughput per user*

Fig. 3 depicts the variation of multicasting throughput per user obtained by using asymptotic analysis, Monte-Carlo approach and by approximately exact analysis approach along with SNR for different number of users operating over double Nakagami fading channel. It is observed that the multicasting throughput increases with increase in SNR levels. When the number of users is large the increase in multicasting throughput per user is less as compared to that with smaller number of users. It is obvious from the plot in Fig. 3 that the asymptotic approximation results are in close agreement

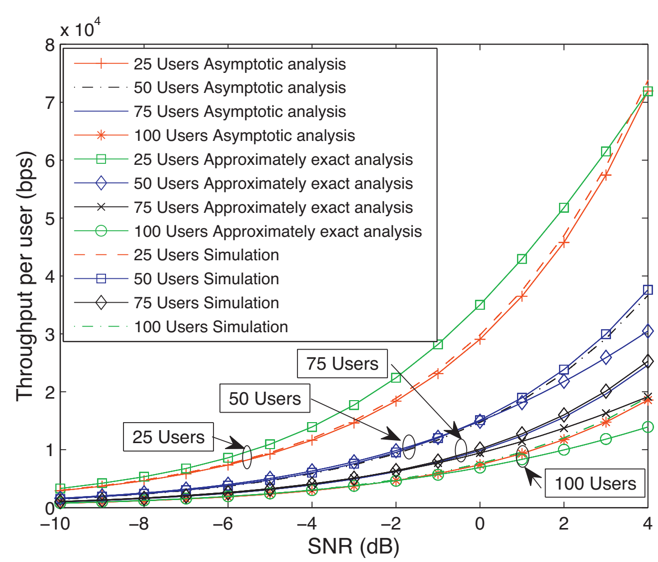

Fig. 3. Multicasting throughput per user versus SNR by varying number of users for communications operating under double Nakagami fading channel.

Table 1
System parameters.

Parameter	Value	Parameter	Value
P_{FLT}	2.5 mW	*Freq.*	2.5 GHz
P_{DAC}	15.5 mW	n	0.47
P_{SYN}	50.0 mW	N_{figure}	10 dB
P_{MIX}	30.3 mW	G_tG_r	5 dBi
$\sigma^2 = N_0/2$	−174 dBm/Hz	L_{margin}	40 dB

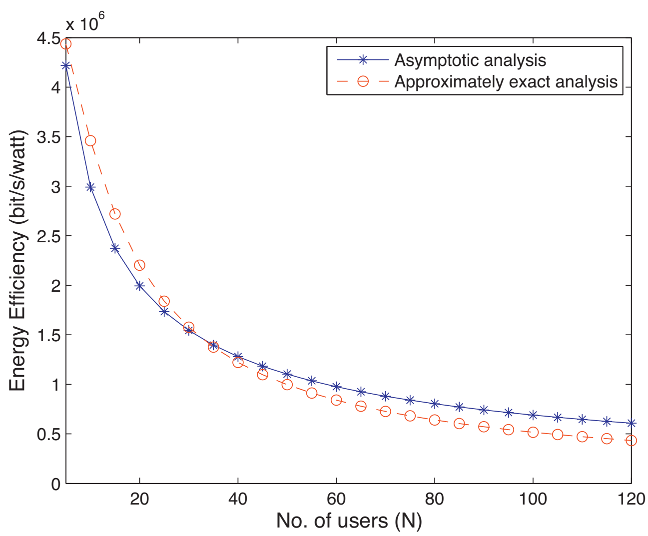

Fig. 4. Energy efficiency versus number of users for communications operating under double Nakagami fading channel.

with the results of approximately exact analysis and the simulation results. Therefore, our results obtained by asymptotic analysis using EVT stand validated.

3.3. Energy efficiency

Table 1 shows the list of important simulation parameters used for energy efficiency computations. Fig. 4 depicts the variation in energy efficiency obtained by asymptotic analysis using EVT along with varying number of users for communications operating over double Nakagami fading channel. It is observed that the energy efficiency decreases with increase in number of users. With the initial increase in number of users, the decrease in energy efficiency is more rapid. Here also, results obtained with asympototic approximation are in close match with that of the approximately exact and simulation approaches.

4. Conclusion

In this work a detailed analysis of multicast throughput and energy efficiency for WSN communications operating under the assumption of independent and identically distributed channels over double Nakagami fading was performed by using Extreme value theory to quantify the stochastic behavior of a process. We used the cumulative distribution function of the SNR of double Nakagami fading to obtain asymptotic expressions for minimum scaled SNR. The distribution of minima extreme was shown to converge to Weibull distribution and then subsequently, the effects of minimum scaled SNR on the multicasting throughput were obtained. We also studied the behavior of multicasting throughput per user by varying the number of users, severity of fading and SNR. We also obtained the results in context of energy efficiency. The results obtained by asymptotic analysis using EVT were found to be in close match with those of approximately exact analysis approach and Monte-Carlo simulation approach. Some of the important conclusions of our study are:

- The asymptotic approach using EVT because of its convenience and ease in handling problems can be applied to broader range of channel fading distributions. The application of EVT provides an easy alternative to otherwise intractable challenges.
- Multicasting throughput per user decreases rapidly with increase in the number of users when the number of users is small, after that variation is slow with further increase in the number of users.
- Multicasting throughput per user also decreases with increase in severity of fading and increases with increase in SNR levels. Results also conclude that with varying SNR from low to high, increase in multicasting throughput for large number of users is less than as that for small number of users.
- The energy efficiency decreases with increase in number of users. With initial increase, the fall is rapid, whereas with further increase, the energy efficiency slowly decreases.
- A close match between the results obtained from asymptotic approximation and recently reported approximately exact analysis approach and Monte-Carlo simulation approach demonstrates that the proposed asymptotic approximation is sufficiently valid.

These studies can be extended for other fading models of interest and may facilitate design of high throughput friendly and energy-efficient WSN architectures even in the presence of constrained environments.

Appendix A. Extreme value theory [23,32]

In some problems, we are interested in how a particular order statistics behaves for larger population. In those cases, when either the parent distribution or the sample size is not fully known, the solution becomes intractable, then analyzing the limiting distributions if they exist, may provide useful insights. According to theory of extreme values, the largest or smallest value from a set of independent identically distributed random variables tends to an asymptotic distribution that only depends on the tail of the distribution of the basic variable. Let $X_{(1)}, X_{(2)}, X_{(3)} \ldots X_{(n)}$ be the ordered set of the same variables, then the distribution of $X_{max} = X_{(n)}$ is given by

$$F_{X_{max}}(x) = [F_X(x)]^n \qquad (A.1)$$

If $H_n = (X_{(n)} - b_n)/a_n$, where $a_n > 0$ denotes a scaling constant and b_n is a location constant, then this limiting distribution must be one of the three following types:

$$H_1(y;\gamma) = \begin{cases} exp(-y^{-\gamma}), & y > 0; (FRECHET) \\ 0, & y \leq 0. \end{cases} \qquad (A.2)$$

$$H_2(y,\gamma) = \begin{cases} exp[-(-y)^{\gamma}], & y < 0; (WEIBULL) \\ 1, & y \geq 0. \end{cases} \qquad (A.3)$$

$$H_3(y) = \exp(-\exp(-y)), \quad -\infty < y < +\infty; (GUMBEL) \qquad (A.4)$$

Here, $\gamma > 0$ is a positive constant. The existence of these three asymptotic forms of the extreme distribution relies on the stability postulate, which states that if X has an extreme value distribution, the maximum of n independent observations of X has the same distribution, but with different location and scale parameters. Thus, the solution of

$$[F_X(x)]^n = F_x\left(\frac{x-b_n}{a_n}\right) \tag{A.5}$$

where a_n and b_n are functions of n, yields all the possible limiting forms of $F_X(x)$ as $n\to\infty$.

Appendix B. Smallest value distribution [23,32]

In many cases, we are interested in the asymptotic distributions of the smallest value. The distributions of the largest and smallest values are related by the principle of symmetry. Using the principle of symmetry, the asymptotic distribution of the smallest value of a random variable can be determined from the distribution of the largest value by reversing the sign and taking the complimentary probabilities.

We get the following three asymptotic distributions of the smallest value as follows

$$L_1(y) = \begin{cases} 1-\exp[-(-y)^{-\gamma}], & y<0; (FRECHET) \\ 1, & y\geq 0. \end{cases} \tag{B.1}$$

$$L_2(y) = \begin{cases} 1-\exp(-y^{\gamma}), & y>0; (WEIBULL) \\ 0, & y\leq 0. \end{cases} \tag{B.2}$$

$$L_3(y) = 1-\exp(-\exp(y)), \quad -\infty<y<+\infty; (GUMBEL) \tag{B.3}$$

The sufficient conditions for convergence of L_1, L_2 and L_3 are available in [23,32]. We mention the condition of our interest for Weibull distribution in the theorem of Appendix C.

Appendix C. Theorem [23]

Theorem 1. *Let Z_i, $i=1, \ldots, N$ be independent and identically distributed random variables with CDF $F_Z(z)$. Let $\alpha(F)$ be defined as*

$$\alpha(F_Z) = Inf\{z : F_Z(z) > 0\} \tag{C.1}$$

If $\alpha(F) > -\infty$ and the function

$$F_Z^*(z) = F_Z\left(\alpha(F) - \frac{1}{z}\right); \quad z<0 \tag{C.2}$$

satisfies

$$\lim_{t\to-\infty} \frac{F_Z^*(tz)}{F_Z^*(t)} = z^{-\gamma}, \quad \gamma>0 \tag{C.3}$$

then, there exist constants c_N and d_N such that $(Z_{min,N}-c_N)/d_N$ converges to Weibull distribution for minima, where the normalizing constants are $c_N=\alpha(F)$ and $d_N = F_Z^{-1}\left(\frac{1}{N}\right) - \alpha(F)$.

Weibull distribution (minima) is $L_2(y) = 1-\exp(-y^{\gamma})$ if $y>0$ and 0 otherwise. It is equivalent to exponential distribution for $\gamma=1$.

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