

On Channel Estimation of OFDM-BPSK and -QPSK over Generalized Alpha-Mu Fading Distribution

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Received January 9, 2010; revised February 11, 2010; accepted March 18, 2010

Abstract

This paper evaluates the performance of OFDM-BPSK and -QPSK system in α - μ distribution. A fading model which is based on the non-linearity present in the propagation medium is utilized here for generation of α - μ variants. Different combinations of α and μ provides various fading distributions, one of which is Weibull fading. Investigations of channel estimation schemes gave an idea of further reducing the BER as to improve the performance of OFDM based systems. In flat fading environment, channel estimation is done using phase estimation of the transmitted signal with the help of trained symbols. Final results show the improvement in BER. However, the amount of results improved depends upon the amount of trained symbols. The more trained symbols will result into more improved BER.

Keywords: OFDM, Fading Distribution, Weibull Fading, Nakagami Fading, Channel Estimation, Training Symbols

1. Introduction

In recent years, OFDM have been studied very widely and deeply in wireless communication systems because of bandwidth efficiency and its robustness to channel fading and Inter Symbol Interference (ISI). OFDM system is capable of mitigating a frequency selective channel to a set of parallel fading channels, which need relatively simple processes for channel equalization.

There exists a large number of distribution schemes to describe the statistics of mobile radio signal. A key assumption in the theoretical explanation of the Rayleigh, Rician, Nakagami and Weibull distribution was that the statistics of the channel do not change over the small (local) area under consideration. However, to describe the long-term signal variations lognormal distribution is being used [1]. These distributions are helpful in precise designing of wireless systems to make the systems more robust to noise.

Rayleigh and Rician fading channels have already been studied and employed in OFDM systems in frequency selective and flat fading environment. Nakagami-m distribution is another useful and important model to characterize the fading model. A threshold value of m is calculated for both frequency and flat fading environment in [2]. There exist many other fading models in literature, which have been proposed for better fitting of

data while aiming at non-linearity of channel. So our motivation behind this paper to explore the non-linear fading environment. One of the interesting models that we could find in literature is α - μ distribution [3], which provides the generalized model for fading distribution. Depending upon the value of α and μ , this model can be utilized for the generation of Nakagami-m and Weibull variants. However, it also treats One-Sided Gaussian, Rayleigh and Negative Exponential distributions as its special cases. The generalized fading model using three parameter generalized gamma distribution describing all forms of multipath fading and shadowing in wireless systems is analyzed in [4].

This paper is organized as follows: In Section 2, OFDM system model is discussed. Section 3, describes the generalized model of α - μ distribution. Flat fading channel model to use in OFDM systems is described in Section 4. In Section 5, channel estimation technique is discussed. The analysis of OFDM system without estimation has been done in Section 6, while results with estimation have been presented in Section 7. Finally Section 8 concludes the paper.

2. OFDM Model

A Complex base band OFDM signal with N sub-carriers,

is expressed as

$$s(t) = \sum_{k=0}^{N-1} D_k e^{j2\pi k f_0 t} \quad 0 \leq t \leq T \quad (1)$$

For each OFDM symbol, the modulated data sequences are denoted by $D(0), D(1), \dots, D(N-1)$. Here, f_0 denote the sub-carriers spacing and is set to $f_0 = 1/T$, the condition of orthogonality. After IFFT, the time-domain OFDM signal can be expressed as:

$$S(n) = \frac{1}{N} \sum_{k=0}^{N-1} D_k e^{\frac{j2\pi k f_0 n}{N}} \quad (2)$$

After IFFT, the modulated signal is up-converted to carrier frequency f_c and then the following signal is produced and transmitted through channel:

$$x(t) = \text{Re} \left\{ \sum_{k=0}^{N-1} D_k e^{j2\pi k (f_0 + f_c) t} \right\} \quad 0 \leq t \leq T \quad (3)$$

$x(t)$ represents the final OFDM signal in which sub-carriers shall undergo a flat fading channel.

3. The $\alpha - \mu$ Distribution

The $\alpha - \mu$ distribution is a general fading distribution that can be used to represent various fading model. This distribution deals with non-linearity of propagation medium [5]. Fading signal with envelope r , an arbitrary constant parameter $\alpha > 0$ and a root mean value $\hat{r} = \sqrt[\alpha]{E(r^\alpha)}$ shall have its probability density function $p(r)$, which is written as:

$$p(r) = \frac{\alpha \mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp(-\mu \frac{r^\alpha}{\hat{r}^\alpha}) \quad (4)$$

Weibull and Nakagami-m distribution can be easily derived from $\alpha - \mu$ distribution as its special cases.

By setting $\mu = 1$, Equation (4) shall reduce to Weibull probability distribution function as:

$$p(r) = \alpha \beta r^{\alpha-1} \exp(-\beta r^\alpha) \quad (5)$$

where $\beta = \hat{r}^{-\alpha}$.

Here, by varying the value of α different curves of pdf can be plotted.

From Weibull distribution by setting $\alpha = 2$, the Rayleigh distribution can be obtained as:

$$p(r) = \frac{r}{\gamma^2} \exp(-\frac{r^2}{2\gamma^2}) \quad (6)$$

where $\gamma^2 = \hat{r}^2 / 2$.

Now, if we put $\alpha = 1$ in Weibull distribution, it shall reduce to Negative exponential distribution represented as:

$$p(r) = \delta \exp(-\delta r) \quad (7)$$

where $\delta = \hat{r}^{-1}$

So by keeping the value of $\mu = 1$ and varying the value of α it has generated Rayleigh and Negative exponential distribution. Whereas if we keep $\alpha = 2$ and vary the value of μ , we shall be able to represent this $\alpha - \mu$ distribution as Nakagami-m distribution

In such a case

$$p(r) = \frac{2\mu^\mu r^{2\mu-1}}{\Omega^\mu \Gamma(\mu)} \exp(-\mu \frac{r^2}{\Omega}) \quad (8)$$

By setting $\mu = 1/2$, one-sided Gaussian distribution can be obtained as:

$$p(r) = \sqrt{\frac{2}{\pi \hat{r}^2}} \exp(-\frac{r^2}{2\hat{r}^2}) \quad (9)$$

However, for $\alpha - \mu$ distribution the envelope r can be written as:

$$r = \left[\sum_{i=1}^N (x_i^2 + y_i^2) \right]^{1/2} \quad (10)$$

where, x_i and y_i are in-phase and quadrature elements of multipath components represented by symbol i .

It was interesting to find that $\alpha = 2$ in Equation (10) shall reduce to the envelope equation of Rayleigh fading distribution [6] described as:

$$r = \left[\sum_{i=1}^N x_i^2 + y_i^2 \right] \quad (11)$$

Same concept has been shown in Equations (5) and (6) that by putting $\alpha = 2$, Weibull distribution converts to Rayleigh distribution, hence introducing the non-linearity into propagation medium. However, at different values of α different fades can be generated.

4. Channel Model

In this paper, the sub-channel spacing is equal to inverse of time period, so that the produced parallel fading sub-channels have flat fading characteristics.

Here $\alpha - \mu$ distribution has been utilized for generation of Weibull distribution by setting $\mu = 1$ and varying the value of α .

In flat fading environment, the base-band signal at the input of receiver $y(t)$ is as described as follows:

$$y(t) = x(t) * r(t) + n(t) \quad (12)$$

where, $x(t)$ denotes the base-band transmitted signal, $r(t)$ is the Weibull distributed channel envelope and $n(t)$ is the additive white Gaussian noise with zero mean.

5. Channel Estimation

Channel estimation in frequency selective has different approach then compared with flat fading environment. A comparative study using Minimum Mean Squared Error (MMSE) and Least square (LS) estimator in frequency selective fading environment has been presented in [7]. The channel estimation based on comb type pilot arrangement is studied using different algorithms by bahai *et al.* [8]. A novel channel estimation scheme for OFDMA uplink packet transmissions over doubly selective channels was suggested in [9]. The proposed method uses irregular sampling techniques in order to allow flexible resource allocation and pilot arrangement. In flat fading environment, estimation of the channel using trained sequence of data has been studied and implemented in [10]. He presented the channel estimation in flat fading environment using some trained data. Channel phase was estimated during each coherence time. Then pilot data of some required percentage of data length (referred as training percentage in simulation) is inserted into the source data. It is used to estimate the random phase shift of the fading channel and train the decision to adjust the received signal with phase recover. The results obtained showed the great variation in BER for with and without estimation curves. It is clear from literature reviewed that phase estimation using training symbol can be implemented in flat fading environment to improve the performance of system.

In this paper, we have implemented the above described phase estimation technique in flat fading for Weibull fading distribution on OFDM system.

6. Results without Estimation

OFDM-BPSK and -QPSK signal is simulated in MATLAB environment by choosing total number of sub-carriers 400, IFFT length 1024 by using guard interval of length 256.

The results presented in **Figures 1** and **2** are simulated by varying the value of α and keeping $\mu = 1$. Here, the BER values have been obtained for varying α over a range of 1 to 7, however, improvement in BER was not significant for higher values of α , So range has been

kept from 1 to 7, both for OFDM-BPSK and -QPSK system.

From the simulations, it has been verified that the results for $\alpha = 2$ are same that are obtained by using Rayleigh fading distribution.

It has been observed that if we plot BER curves by changing the value of μ there is no change in these curves. This is because of fact that in the envelope Equation 10, the variable parameter is α which varies the fading variants and μ has no role to change this fading envelope and hence no change in BER values.

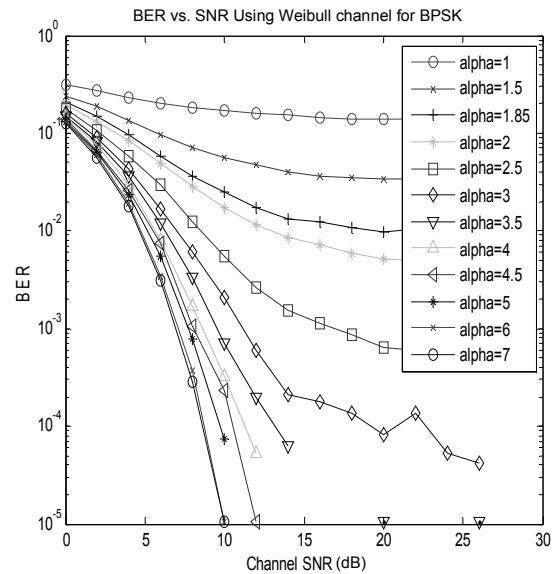


Figure 1. BER vs. SNR for OFDM-BPSK system.

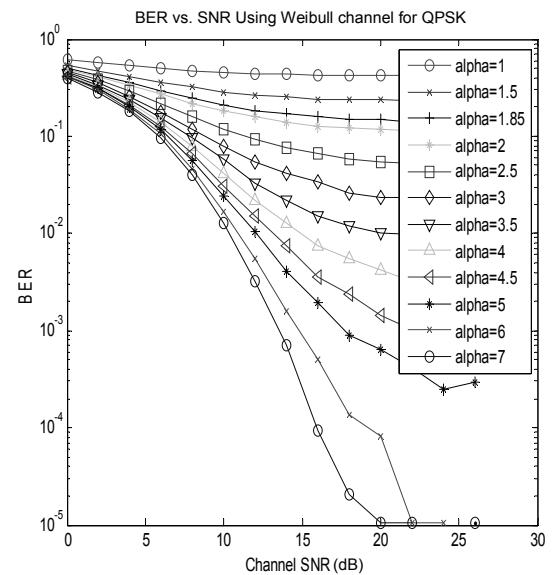


Figure 2. BER vs. SNR for OFDM-QPSK system.

To explore the other side of $\alpha - \mu$, by keeping the value of α fixed and varying the value of μ , we are able to have BER curves for other distributions. In **Figure 3**, Negative exponential distribution has been plotted as special case where $\alpha = 1$ and μ can vary. Here, it has been plotted for fixed value of 1. Rayleigh distribution is having $\alpha = 2$ and μ can vary. Here, it has been plotted with $\mu = 1$. One sided distribution has been plotted with $\alpha = 2$ and $\mu = 1/2$.

BER varies in the range of 10^{-1} to 10^{-5} for OFDM-BPSK and -QPSK for SNR of 0 to 25 dB. In case of OFDM-BPSK the BER value of 10^{-5} is obtained at SNR of 10dB. However, for -QPSK case the BER of 10^{-5} is obtained at SNR of 20 dB.

Comparison between Negative exponential value, Rayleigh and one sided distribution results clearly reveals the fact that in $\alpha - \mu$ distribution the variation in value of α can change the value of BER, however by changing the value of μ , there is no impact upon the BER. Results obtained without estimation technique has been presented in [11].

7. Results with Estimation

Trained symbols are added to source signal as discussed in Section 5. The percentage of such symbol may be varied depending upon the system response to the trained sequence. We have analyzed the results for various percentage values of trained sequence. We have plotted new graphs with value of $\alpha = 7$ and varying value of training sequence over the range from 10% to 50%. Results

for OFDM-BPSK and -QPSK have plotted in **Figures 4** and **5** respectively.

In depth analysis of these graphs shows that BER decreases, if the training percentage is increased. In **Figure 5**, if we evaluate the reading obtained at SNR = 10 db, BER has decreased from 0.0208 to 0.001 for training percentage of 10 to 50. This means, for the same value of SNR and α , different training percentage has resulted into different values of BER. More trained sequence will results into lesser errors. The same has been depicted from **Figure 4**.

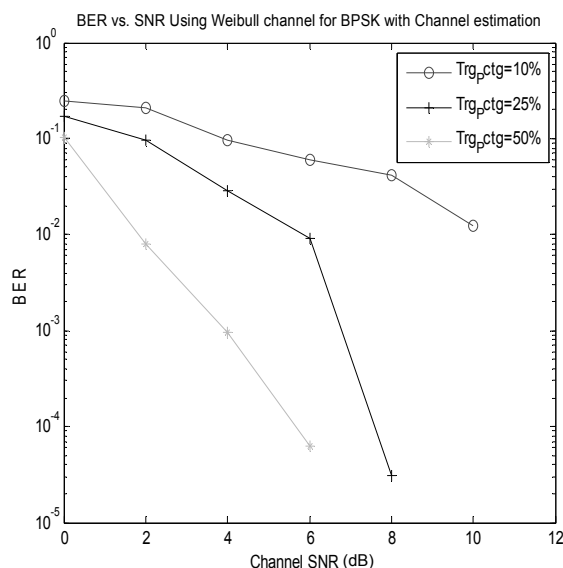


Figure 4. BER vs. SNR for OFDM-BPSK with channel estimation.

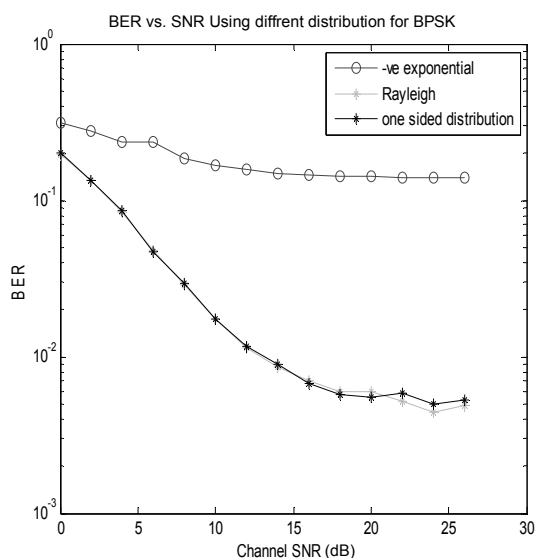


Figure 3. BER vs. SNR for negative, rayleigh and exponential distributions.

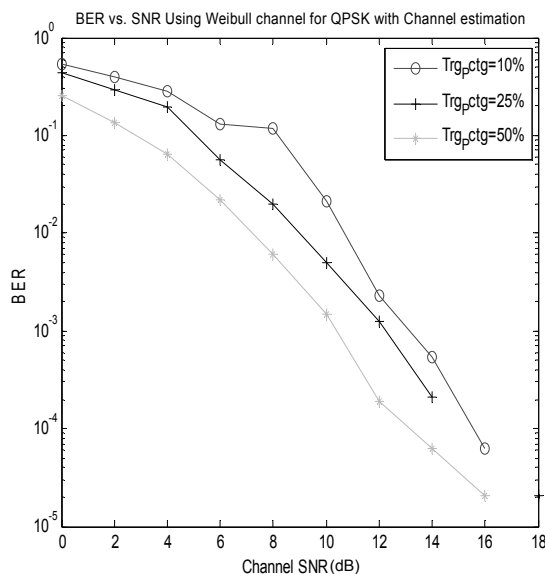


Figure 5. BER vs. SNR for OFDM-QPSK with channel estimation.

8. Conclusions

This paper, presents performance analysis of OFDM system with generalized fading model of $\alpha - \mu$ distribution with and without estimation. The non-linearity added in propagation medium has been clearly shown in simulated results, since the BER has significantly reduced by varying α from 1 to 7. However, higher values of α can be used for further reductions in BER. It is clear from the simulations that the result shows significant improvement by applying the phase estimation using trained symbols.

9. References

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