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Minimization of self-steepening of ultra short higher-order soliton pulse at 40 Gb/s by the optimization of initial frequency chirp

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Abstract

In this paper, the decay of first- and second-order ultra short pulses of the order of 50 fs due to self-steepening (SS) effect has been numerically investigated for a 40 Gb/s optical soliton system including the impact of third-order dispersion (TOD). It has been observed that the prechirp (both positive and negative) in the pulse can counteract the SS effect and improve compensation performance for the distortions .The critical positive prechirp value is found to be 1.4 and the negative prechirp is 1.2, beyond which the soliton pulse is unstable.
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1. Introduction

Optical fiber solitons are considered to be one of the most attractive techniques for future high-speed and long-distance optical communications because it can keep its waveform through the interplay between periodically distributed dispersion and fiber nonlinearity. Although the solitons have been successfully exploited to achieve long transmission distance at a high bit rate, the practical implementation is limited by the number of factors to maintain the exact balance between nonlinearity and dispersion. For the propagation of high data rate ultra short optical pulses with the pulse width $T_0 < 100$ fs, it is imperative to include the higher-order dispersive and nonlinear effects. An important higher-order nonlinear effect is self-steepening (SS), which results from the intensity dependence of the group velocity. It leads to an asymmetry in the self-phase modulation (SPM) broadened spectra [1–3]. The influences of the retarded nonlinear response and the SS effect on solitons propagating in an optical fiber have been studied, and it is found that the retarded nonlinear response leads to self-frequency shift [1,4] and SS of an optical pulse leads to steeper trailing edge of the pulse [5–7], which could cause an optical wave shock in the absence of group velocity dispersion (GVD). Both the retarded nonlinear response and the SS effects result in higher-order soliton decay.

Dong and Liu [6] derived a soliton resulting from the combined effect of higher-order dispersion, SS and nonlinearity in an optical fiber with a non-vanishing boundary condition for the generalized nonlinear Schrödinger equation (NLSE) under the condition

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$\beta_2/\beta_3 = \omega_0/2$ (β_2 is the second-order dispersion, β_3 is the third-order dispersion (TOD) and ω_0 is the carrier frequency) neglecting the terms concerned with loss and retarded nonlinear response. It was reported that in the normal dispersion regime, dark solitons exist; however, in the anomalous dispersion regime, bright solitons exist superimposed on the continuous wave background, and especially in the long wave limit rational solitons also exist.

Chen et al. [7] investigated the propagation of ultra short pulses along the slow group velocity fiber. A more generalized NLSE as the superposition of monochromatic waves was derived and the propagations of 2.5-fs fundamental and 5-fs second-order solitons were numerically studied. It was found that, for a slow group velocity fiber, the magnitude of time shift is related with the group velocity and the more generalized NLSE is more suitable than the conventional NLSE. When the pulse slowed down to 50% of normal group velocity (c/n_0), the effect of the higher nonlinear terms was significant. In another paper, Wang and Wang [8] presented the numerical algorithm for solving coupled extended NLSEs including higher-order dispersion, retarded nonlinear response and SS terms. The numerical results showed that the influence of the retarded nonlinear response and SS effect on the coupling dynamics in a nonlinear directional coupler is strongly dependent on the input peak power, input pulse width, and product of the dispersion length and the coupling coefficient ($L_D\kappa$). In the case of $L_D\kappa \gg 1$, the pulse coupling obeys Jensen's equation as long as the input pulse width is broader than 250 fs and the coupler is far from zero dispersion. Both the retarded nonlinear response and the SS effect could be ignored if the normalized amplitude of the input pulse was less than 0.5.

The exact analytical solution of the optical soliton equation with higher-order dispersion and nonlinear effects by the method of separating variables was obtained by Fu-quan Dou et al. [9]. The stability of the solitary wave solutions for the optical soliton equation was discussed by using the Liapunov direct method. Lin and co-workers. [10] investigated the prechirped femtosecond pulse amplification, compression and SS processes in a homemade large-mode-area erbium-doped fiber amplifier with ultra short cavity length. A total pulse width compression ratio of up to 40 and a maximum peak-power amplification ratio of >20 dB could be simultaneously achieved. An SS-effect-induced blue-side spectral stretch by 1.3 THz was observed and elucidated. Ramprasad et al. [12] analytically studied the one-dimensional NLSE using the scalar and vector approach. The propagation characteristics of the pulses in the fiber media were analyzed and verified by simulation considering various nonlinear effects like stimulated Raman scattering and SS effects using the split-step Fourier transform (SSFT) method of simulation based on partitioning of fibers into spans.

It is apparent from the above literature that the influence of SS effect in the higher-order solitons has been comprehensively studied in the past decade. However, the techniques to counteract the SS effect, which is the major cause of soliton decay, are still being explored and accessed quantitatively. Bu and Wang [11] presented compensation of the SS effects in an optical fiber communication system using the midway optical phase conjugation method.

In this paper, we analyze the decaying of first- and second-order soliton pulses having pulse width 50 fs due to the SS effect. For a 40 Gb/s optical soliton system including the impact of TOD, the SS effect is examined with the emphasis on its minimization by optimizing the initial frequency chirp of the pulse. The investigations have been carried out without and with positive and negative values of prechirp at the optical source itself. Here, the theoretical aspect of the SS effect for femtosecond pulses have been discussed in Section 2, the results of simulation are reported in Section 3 and the conclusions are summarized in Section 4.

2. Theory

As the ultra short pulse having width $T_0 < 1$ ps propagates inside the fiber, it becomes asymmetric with its peak shifted towards the trailing edge, resulting in soliton decay. Since the spectral width $\Delta\omega$ of such pulses becomes comparable to the carrier frequency ω_0 and the spectrum is wide enough ($\geqslant$THz) that the Raman gain can amplify the low-frequency components by transferring to the high-frequency components of the same pulse, the pulse spectrum shifts towards the red side and the trailing edge becomes steeper and steeper with the increase in distance. The steeper trailing edge of the pulse implies larger spectral broadening on the blue side than on the red side, as SPM generates blue components near the trailing edge. Physically, the group velocity of the pulse is intensity dependent such that the peak moves at a lower speed than the wings; it is delayed and appears to be shifted towards the trailing edge.

The propagation of ultra short optical soliton pulses with widths $\leqslant 100$ fs inside a single-mode fiber is described by the generalized nonlinear Schrödinger wave equation taking into consideration the higher-order dispersive and nonlinear effects as [1,5]

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \beta_1\frac{\partial A}{\partial T} - \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{1}{6}\beta_3\frac{\partial^3 A}{\partial T^3} + i\gamma \times \left[|A|^2A + \frac{2i}{\omega_0}\frac{\partial}{\partial T}(|A|^2A) - T_R A\frac{\partial}{\partial T}(|A|^2)\right] \quad (1)$$

where A is the slowly varying complex envelop of the soliton pulse, z is distance propagated by the pulses and T is the time measured in a frame of reference moving with the pulse at the group velocity v_g ($T = t - z/v_g$), α is the absorption coefficient, β_1, β_2 and β_3 are, respectively, the first-, second- and third-order dispersions. T_R is the response time related to the slope of the Raman gain [4], the term having the factor $1/\omega_0$ is related to the SS effect [5,6,8] and nonlinearity coefficient γ that results in SPM is related to the nonlinear refractive index n_2 by

$$\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}} \tag{2}$$

where A_{eff} is the effective core area of the fiber.

In terms of normalized amplitude U with ξ and τ representing the normalized distance and time variables, the propagation Eq. (1) becomes [1,5]

$$\mathrm{i}\frac{\partial U}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} + \mathrm{i}\delta_3 \frac{\partial^3 U}{\partial \tau^3} - N^2\left[|U|^2 U + \mathrm{i}s\frac{\partial}{\partial \tau}(|U|^2 U) - \tau_R U \frac{\partial}{\partial \tau}(|U|^2)\right] \tag{3}$$

where the pulse is assumed to propagate in the region of anomalous GVD ($\beta_2 < 0$) and the fiber loss is negligible setting $\alpha = 0$. The parameters δ_3, s and τ_R govern, respectively, the effects of cubic dispersion, SS and retarded nonlinear response. Their explicit expressions are: where

$$U = \frac{A}{\sqrt{P_0}}, \quad \xi = \frac{z}{L_D}, \quad \tau = \frac{T}{T_0} \tag{4}$$

and

$$\delta_3 = \frac{\beta_3}{6|\beta_2| T_0}, \quad s = \frac{2}{\omega_0 T_0}, \quad \tau_R = \frac{T_R}{T_0} \tag{5}$$

P_0 is the peak power, T_0 is the width of the incident pulse, L_D is the dispersion length and another parameter N characterizes the soliton order that provides the measure of the strength of the nonlinear response compared to the fiber dispersion, and is defined as [1]

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \tag{6}$$

where the dispersion length L_D and the nonlinear length L_{NL} provide the length scales over which the dispersion or nonlinear effects become important for pulse evolution along a fiber of length L and are given by

$$L_D = \frac{T_0^2}{|\beta_2|}, \quad L_{NL} = \gamma P_0 \tag{7}$$

Since all the three parameters δ_3, s and τ_R vary inversely with the pulse width, therefore are negligible for $T_0 > 1$ ps, however they become appreciable for femtosecond pulses.

To consider the effect of SS only, governed by the parameter s, $\tau_R = 0$ have been substituted in Eq. (3), pulse evolution is thus governed by

$$\mathrm{i}\frac{\partial U}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} + \mathrm{i}\delta_3 \frac{\partial^3 U}{\partial \tau^3} - N^2\left[|U|^2 U + \mathrm{i}s\frac{\partial}{\partial \tau}(|U|^2 U)\right] \tag{8}$$

To investigate the soliton propagation in optical fibers, the system is numerically simulated using the split-step Fourier method with the initial envelope of the soliton given by $U(0, \tau) = N\,\mathrm{sech}(\tau)$ for $N = 1$ and 2. The temporal evolution of the first- and second-order soliton pulses over 10 soliton periods where soliton period z_0 is related to the initial pulse width and is given as

$$z_0 = \frac{\pi}{2} L_D = 0.332 \frac{\pi}{2} \frac{\tau^2}{|\beta_2|} \tag{9}$$

SS of the trailing edge of the pulse eventually creates an optical shock on the leading edge of the pulse. The critical distance for the formation of the shock can be given as [3]

$$z_s = \left[\frac{e}{2}\right]^{1/2} \frac{L_{NL}}{3s} \approx 0.43 \frac{L_{NL}}{s} \approx \frac{1.35 T_0}{\gamma P_0 T} \tag{10}$$

For picosecond pulses with $T_0 \approx 1$ ps and $P_0 \approx 1$ W, $z_s \approx 100$ m, but for femtosecond pulses with $T_0 < 100$ fs, $P_0 > 1$ kW, z_s becomes less than 1 m. As a result, significant SS of the pulse occurs over a few centimeter long fibers.

To minimize the SS effect, the effect of initial frequency chirp was included in the input pulse amplitude and the expression of the chirped pulse is given by

$$U_{\text{chirp}}(0, \tau) = N\,\mathrm{sech}(\tau) \exp(-\mathrm{i}C\tau^2/2)$$

where C is the chirp parameter .The quadratic form of the phase variation in the equation corresponds to a linear chirp such that the optical frequency increases with time (up-chirp) for the positive values of C. The system has been simulated for second-order soliton pulses by considering the small positive and negative linear prechirp and the results were obtained by varying the value of positive chirp in the range of 0–1.6 and negative chirp from 0 to −1.4.

3. Results and discussion

The optical soliton system has been numerically simulated for 10 soliton periods by using SSFT method. The various simulation parameters considered are bit rate = 40 Gb/s, samples per bit = 1024, central frequency of the pulse = 193.1 THz, pulse width $T_{\text{FWHM}} = 2\ln(1 + \sqrt{2})\ T_0 = 50$ fs, $T_0 = T_{\text{FWHM}}/1.763 = 28.3 \approx 30$ fs. The bit sequence in the user defined bit sequence generator is "…0100…".

The peak power necessary to launch the Nth order soliton is given by Eq. (6), thus power $P_0 = 232.4434\,\text{W} \approx 0.25\,\text{kW}$ for ($N = 1$) and $P_0 \approx 1\,\text{kW}$ for ($N = 2$).

The hyperbolic secant pulse is launched over a 10 m long nonlinear dispersive fiber. The dominant parameters of the fiber are $\beta_2 = -1\,\text{ps}^2/\text{km}$, $\beta_3 = 0.0744876\,\text{ps}^3/\text{km}$, $A_{\text{eff}} = 93\,\mu\text{m}^2$, $n_2 = 3\times10^{-20}\,\text{m}^2/\text{W}$, max nonlinear phase shift = 20 mrad, $\omega_0/c = 2\pi/\lambda = 2\pi/1.5\times10^{-6}\,\text{m}^{-1}$ and nonlinear refractive index is given by Eq. (2):

$$\gamma = \frac{n_2\omega_0}{cA_{\text{eff}}} \approx 3.3\,\text{W}^{-1}\,\text{km}^{-1}$$

The investigations have been carried out for varying values of positive and negative chirp to analyze the performance of optical soliton transmission as the initial chirp can be detrimental to the soliton propagation simply because it disturbs the exact balance between the GVD and SPM [1]. Recently, phase modulation prior to launch, which is similar to chirp, has been used as a countermeasure against the deleterious effects of fiber nonlinearity [14].

3.1. Case I ($N = 1$, fundamental-order soliton pulse without initial chirp)

The pulse shapes of fundamental-order soliton ($N = 1$) induced by the SS effect ($s = 0.2$, $\tau_R = 0$) over 10 soliton periods have been shown in Fig. 1. The SS-induced temporal shift is clearly visible where pulse shapes for soliton periods at $z/z_0 = 0$, 4, 8 and 10 are plotted for the input $u(0,\tau) = \text{sech}(\tau)$. The pulse evolves asymptotically resulting in the change of group velocity and hence the shift in the peak. It is clearly observed in the figure that the peak of the pulse shifts towards the trailing edge by ~20%, as the peak moves slower than the wings for $s \neq 0$, it is delayed and appears shifted towards the trailing side. Although, the pulse broadens slightly with propagation by ~30% at $z/z_0 = 10$, it nevertheless maintains its soliton nature.

As we know [2], SS creates an optical shock on the trailing edge in the absence of GVD effects, which is due to the intensity dependence of the group velocity. The GVD, however, dissipates the shock and smoothens the trailing edge considerably, although, SS still manifests through the shift in the pulse center. For fundamental-order soliton the steepening of the trailing edge is not very prominent and the peak power also remains the same. The small shock near the leading edge of the pulse is however observed, which is due to the TOD term [5].

3.2. Case II ($N = 2$, second-order soliton pulses without initial chirp)

Fig. 2 shows this behavior for a second-order soliton ($N = 2$) by displaying the temporal evolutions for $s = 0.2$ over 10 soliton periods. It is evident from the figure that the effect of SS on higher-order solitons is remarkable, which leads to breakup of such solitons into their constituents, a phenomenon referred to as soliton fission or soliton decay [2,6]. In particular, second-order soliton decays into two soliton pulses. For this value of s, the two solitons have separated from each other within a distance of four soliton periods and continue to move apart with further propagation inside the fiber. A qualitatively similar behavior occurs for smaller values of s except that a longer distance is required for the breakup of solitons. In the absence of SS ($s = 0$), the two solitons form a bound state because both of them propagate at the

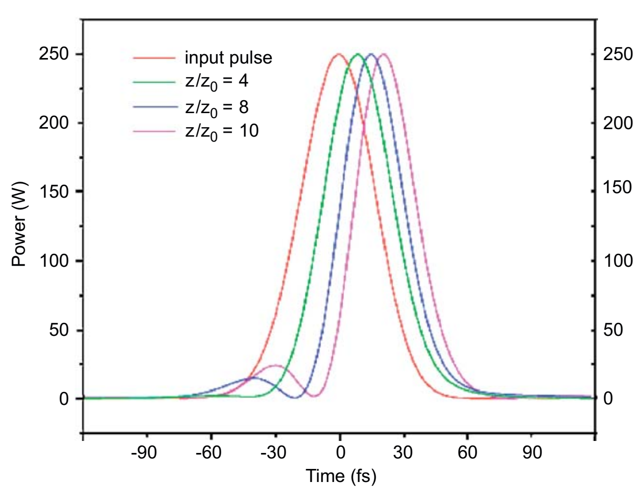

Fig. 1. Temporal evolution of fundamental-order soliton ($N = 1$) induced by self-steepening effect ($s = 0.2$) over 10 soliton periods.

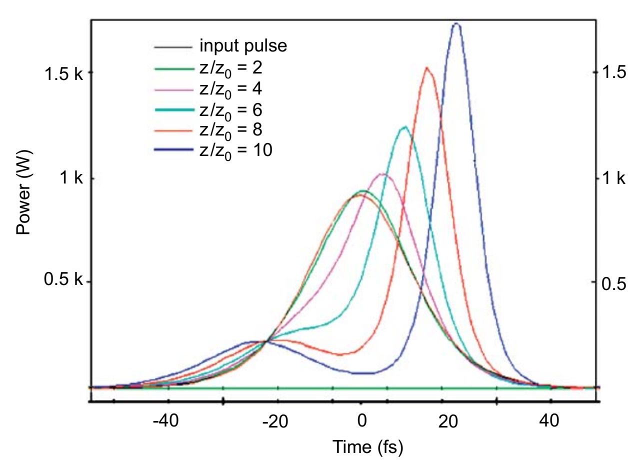

Fig. 2. Temporal evolution of second-order soliton ($N = 2$) induced by self-steepening effect ($s = 0.2$) over 10 soliton periods.

same speed. The effect of SS is to break the degeneracy so that the two solitons propagate at different speeds. As a result, they separate from each other, and the separation increases almost linearly with the distance [6]. The ratio of the peak heights of the two pulses is about 7. It is observed that the soliton peak shifts towards the trailing edge and the pulse steepens at the trailing edge. It can also be noticed that the peak power increases from 1 kW for $z/z_0 = 2$ to almost 1.75 kW for $z/z_0 = 10$ and the soliton no longer maintains its shape. The observations are in good agreement with the results reported in [1,6].

3.3. Case III ($N = 2$, second-order soliton pulses with initial chirp)

Prechirping is one of the important techniques used to employ pre-compensation where appropriate phase modulation of the light carrier is carried in order to compensate for the pulse width broadening resulting from the chromatic dispersion and non-linearity of the optical fiber.

As we have seen the SS effect increases with the order of the system and the second-order soliton pulse is more affected by it than the fundamental-order pulse. Thus, the influence of prechirp has been studied on the second-order soliton pulse. In Fig. 3, pulse shapes for second-order soliton ($N = 2$) have been obtained by varying the values of initial positive chirp ranging from 0 to 1.6 for 10 soliton periods. It has been observed that as the chirp value is increased the SS effect is minimized and the soliton pulse restores its shape at input chirp = 1.4 beyond which the pulse is not stable and the pulse energy is shed away to the side lobes. A similar behavior occurs for the variation in the negative values of C from 0 to -1.4 as can be seen in Fig. 4. The stability of a soliton is expected for small values of $|C|$ because solitons are generally stable under weak perturbations. However, a soliton is destroyed if $|C|$ exceeds a critical value C_{cr} [13] because a part of pulse energy is shed away as continuum radiations during the process of soliton formation and it splits into multiple pulses moving outward. The negative critical chirp value is 1.2 and the positive critical chirp value is 1.4. Also, it can be observed that the reduction in the shift of the central peak is more for the negative chirp as compared to the positive chirp, this is because for $N = 2$, the SPM results in spectral narrowing of the optical spectrum for negatively chirped pulses. Taking into account the above observations it is recommended that the optical source with initial chirp greater than 1.2 is not preferable and the optimum value of the chirp is $C = -1.2$ for an ultra short optical soliton transmission.

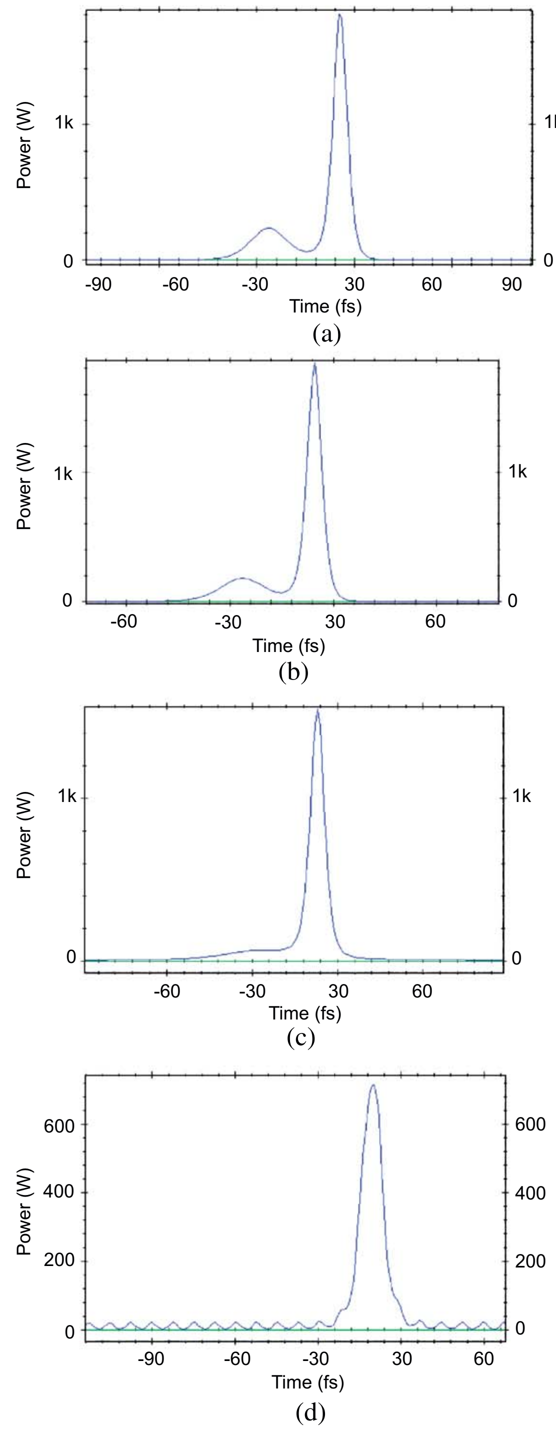

Fig. 3. Pulse shapes for positive prechirp for 10 soliton periods. (a) $C = 0.4$, (b) $C = 0.8$, (c) $C = 1.2$ and (d) $C = 1.6$.

4. Conclusions

In this paper, we demonstrated the decaying of first- and second-order soliton pulses having pulse width 50 fs due to the SS effect for a 40 Gb/s optical soliton system including the impact of third-order dispersion (TOD). The SS effect is minimized by optimizing the initial frequency chirp of the pulse.

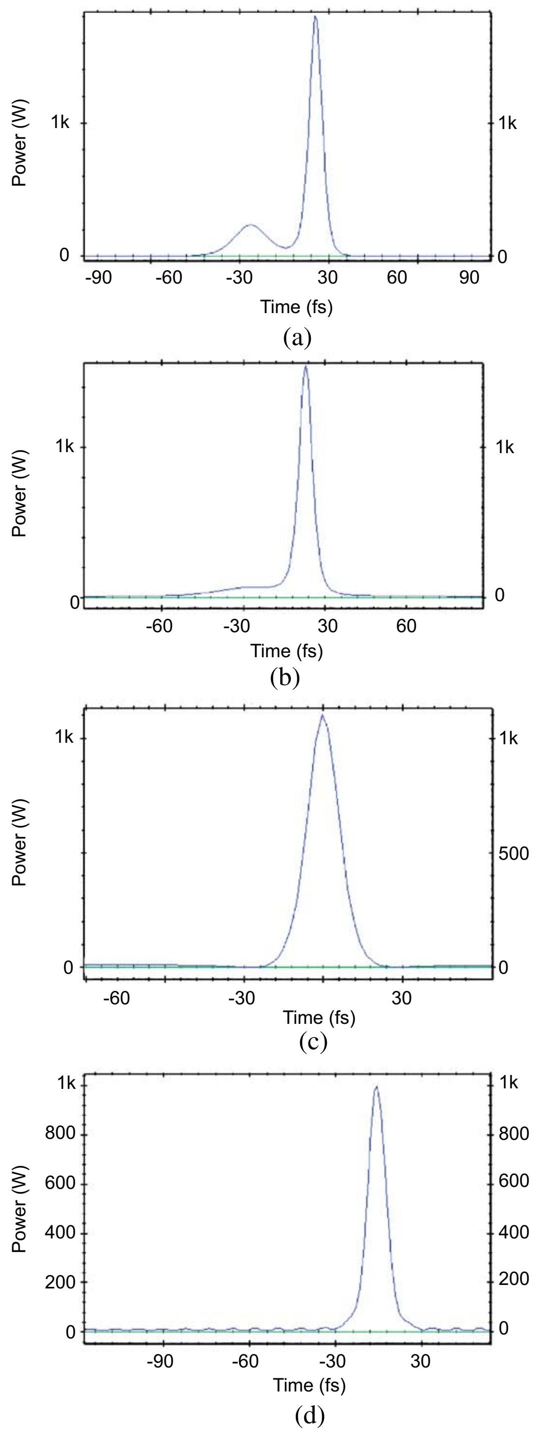

Fig. 4. Pulse shapes for negative prechirp for 10 soliton periods. (a) $C = 0.4$, (b) $C = 0.8$, (c) $C = 1.2$ and (d) $C = 1.4$.

It is concluded that in the fundamental-order ($N = 1$) femtosecond pulses, the SS effect leads to the shift in the central peak towards the trailing edge by ~20%, though there is no decline in the peak power and the pulse maintains its soliton nature. But, for second-order soliton pulses ($N = 2$), the SS effect on the trailing edge is noteworthy and leads to the decay of soliton into two pulses. The soliton peak shifts towards the trailing edge at a much rapid rate and also the peak power increases. It has been observed that the SS effect in the second-order femtosecond pulses can be significantly minimized and the pulse restores its shape for positive and negative values of initial chirp. The critical positive prechirp value is found to be 1.4 and negative prechirp is 1.2 beyond which the soliton pulse is unstable as it sheds away its energy to the side lobes. However, the optimum value of prechirp is $C = -1.2$ for an ultra short optical soliton transmission.

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