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Four-wave mixing analysis in WDM optical communication systems with higher-order dispersion

Amarpal Singh[a,*], Ajay K. Sharma[b], T.S. Kamal[c]

[a] *Beant College of Engineering and Technology, Gurdaspur, Punjab, India*
[b] *National Institute of Technology, Jalandhar, Punjab, India*
[c] *SLIET, Longowal, Punjab, India*

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Abstract

Four-wave mixing (FWM) is one of the major limiting factors in WDM optical fiber communication systems. In this paper, we analyze the individual and combined effect of second-, third-, fourth- and fifth-order dispersion parameters on FWM at different input channel powers and core effective areas, which have not been calculated earlier. FWM power versus channel power graphs for individual and combined effects of dispersion parameters have been presented, and it has been observed that FWM reduces for combined effect of dispersion parameter.

Keywords: FWM; Higher-order dispersion; Core effective area; Channel power

1. Introduction

Four-wave mixing (FWM) (also called four-photon mixing) is one of the major limiting factors in WDM optical fiber communication systems that use the low dispersion fiber or narrow channel spacing. Normally, multiple optical channels passing through the same fiber interact with each other very weakly. However, these weak interactions in glass can become significant over long fiber-transmission distances. The most important is FWM in which three wavelengths interact to generate a fourth [1].

FWM is due to changes in the refractive index with optical power called optical Kerr effect. FWM is a third-order non-linearity in silica fibers that is analogous to inter-modulation distortion in electrical systems. When three electro-magnetic waves with optical frequencies co-propagate through one fiber, they mix to produce a fourth inter-modulation product. In the FWM effect, three co-propagating waves produce nine new optical sideband waves at different frequencies. When this new frequency falls in the transmission window of the original frequencies, it causes severe cross talk between the channels propagating through an optical fiber. FWM occurs when light of three different wavelengths is launched into a fiber; it gives rise to a new wave [2]. This newly generated wave as a result of FWM co-propagates with the originally transmitted signal and interferes with them. It causes severe degradation for the WDM channels. This work is reported in [3] and analyzes the effect of higher-order dispersion parameters on FWM. In this paper, we analyze the effect of higher-order, i.e. up to 5OD, dispersion parameters on FWM power.

Section 2 deals with the theory of FWM and derivation of expression of FWM power including

*Corresponding author.
E-mail address: s_amarpal@yahoo.com (A. Singh).

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higher-order dispersion terms. In Section 3 we plot FWM power versus channel power for all different cases of individual dispersion terms and their combination for different lengths and core effective areas.

2. Theory

The propagation constant in terms of Taylor series can be expanded as

$$\beta = \beta_o + (\omega-\omega_o)\frac{d\beta}{d\omega} + \frac{1}{2}(\omega-\omega_o)^2\frac{d^2\beta}{d\omega^2} + \frac{1}{6}(\omega-\omega_o)^3\frac{d^3\beta}{d\omega^3} + \frac{1}{24}(\omega-\omega_o)^4\frac{d^4\beta}{d\omega^4} + \frac{1}{120}(\omega-\omega_o)^5\frac{d^5\beta}{d\omega^5}\ldots, \quad (1)$$

where $d\beta/d\omega = \tau$, the propagation delay per optical length. Now

$$\beta = \beta_o + (\omega-\omega_o)\tau + \frac{1}{2}(\omega-\omega_o)^2\frac{d\tau}{d\omega} + \frac{1}{6}(\omega-\omega_o)^3\frac{d^2\tau}{d\omega^2} + \frac{1}{24}(\omega-\omega_o)^4\frac{d^3\tau}{d\omega^3} + \frac{1}{120}(\omega-\omega_o)^5\frac{d^4\tau}{d\omega^4}\ldots, \quad (2)$$

$$\Delta\beta = \beta - \beta_o = 2\pi\left[\begin{array}{l}(f-f_o)\tau + \pi(f-f_o)^2\frac{d\tau}{d\omega} + \frac{2\pi^2}{3}(f-f_o)^3\frac{d^2\tau}{d\omega^2}\\ +\frac{\pi^3}{3}(f-f_o)^4\frac{d^3\tau}{d\omega^3} + \frac{2\pi^4}{15}(f-f_o)^5\frac{d^4\tau}{d\omega^4}\ldots\end{array}\right]$$
$$= 2\pi\left[(\Delta f)\tau + \pi(\Delta f)^2\frac{d\tau}{d\omega} + \frac{2\pi^2}{3}(\Delta f)^3\frac{d^2\tau}{d\omega^2} + \frac{\pi^3}{3}(\Delta f)^4\frac{d^3\tau}{d\omega^3} + \frac{2\pi^4}{15}(\Delta f)^5\frac{d^4\tau}{d\omega^4}\ldots\right]. \quad (3)$$

We define here the following dispersion parameters:

$$\beta_2 = \frac{d\tau}{dw} = \frac{\lambda^2}{2\pi c}\frac{\partial\tau}{\partial\lambda} = \frac{\lambda^2}{2\pi c}D$$

is the second-order dispersion parameter,

$$\beta_3 = \frac{d^2\tau}{dw^2} = \frac{\lambda^2}{(2\pi c)^2}\left[\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 2\lambda\frac{\partial\tau}{\partial\lambda}\right] = \frac{\lambda^2}{(2\pi c)^2}[\lambda^2 D_1 + 2\lambda D]$$

is the third-order dispersion parameter,

$$\beta_4 = \frac{d^3\tau}{dw^3} = \frac{\lambda^3}{(2\pi c)^3}\left[\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 6\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 6\lambda\frac{\partial\tau}{\partial\lambda}\right] = \frac{\lambda^3}{(2\pi c)^3}[\lambda^3 D_2 + 6\lambda^2 D_1 + 6\lambda D]$$

is the fourth-order dispersion parameter and

$$\beta_5 = \frac{d^4\tau}{dw^4} = \frac{\lambda^4}{(2\pi c)^4}\left[\lambda^4\frac{\partial^4\tau}{\partial\lambda^4} + 12\lambda^3\frac{\partial^4\tau}{\partial\lambda^3} + 36\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 24\lambda\frac{\partial\tau}{\partial\lambda}\right] = \frac{\lambda^4}{(2\pi c)^4}[\lambda^4 D_3 + 12\lambda^3 D_2 + 36\lambda^2 D_1 + 24\lambda D]$$

is the fifth-order dispersion parameter, where

$$\frac{\partial\tau}{\partial\lambda} = D, \quad \frac{\partial^2\tau}{\partial\lambda^2} = D_1, \quad \frac{\partial^3\tau}{\partial\lambda^3} = D_2, \quad \frac{\partial^4\tau}{\partial\lambda^4} = D_3.$$

As reported in [4], we can neglect $d\beta/d\omega = \tau$, because it produces a phase delay of the carrier signal and has no influence on distortion of the signal. Therefore, by putting the values of β_2, β_3, β_4 and β_5 in Eq. (3) and taking the values only up to the fourth-order derivatives, we obtain the phase matching factor as

$$\Delta\beta = 2\pi^2\Delta f^2\left[\begin{array}{l}\frac{\lambda^2}{2\pi c}D + \frac{2\pi\Delta f}{3}\frac{\lambda^2}{(2\pi c)^2}(\lambda^2 D_1 + 2\lambda D)\\ +\frac{\pi^2}{3}\frac{\Delta f^2\lambda^3}{(2\pi c)^3}(\lambda^3 D_2 + 6\lambda^2 D_1 + 6\lambda D)\\ +\frac{2\pi^3}{15}\frac{\Delta f^3\lambda^4}{(2\pi c)^4}(\lambda^4 D_3 + 12\lambda^3 D_2 + 36\lambda^2 D_1 + 24\lambda D)\end{array}\right],$$

$$\Delta\beta = \frac{\pi\Delta f^2\lambda^2}{c}\left[\begin{array}{l}D + \frac{\Delta f\lambda}{3c}(\lambda D_1 + 2D)\\ +\frac{\Delta f^2\lambda^2}{12c^2}(\lambda^2 D_2 + 6\lambda D_1 + 6D)\\ +\frac{\Delta f^3\lambda^3}{60c^3}(\lambda^3 D_3 + 12\lambda^2 D_2 + 36\lambda D_1 + 24D)\end{array}\right], \quad (4)$$

where Δf is channel spacing, D is fiber chromatic dispersion, $\partial D/\partial\lambda$ is dispersion slope, α is the attenuation factor and c is the velocity of light.

FWM induced cross talk in WDM systems [5–9] and [11], which is given as

$$P_{FWM} = \frac{\eta}{9}D^2\gamma^2 P_i P_j P_k \exp(-\alpha L)\left[\frac{1-\exp(-\alpha L)^2}{\alpha^2}\right],$$

and the FWM efficiency is expressed as

$$\eta = \frac{\alpha^2}{\alpha^2 + \Delta\beta^2}\left[1 + \frac{4\exp(-\alpha L)\sin^2(\Delta\beta L/2)}{(1-\exp(-\alpha L))^2}\right]. \quad (5)$$

The intensity-dependent phase matching factor can be defined [3] as

$$\Delta\beta' = \Delta\beta - \gamma m(P_1 + P_2 - P_3)\left[\frac{\{1-\exp(-\alpha L_{eff})\}}{\alpha L_{eff}}\right], \quad (6)$$

where m is an integer. The phase matching factor changes as the wave propagates along the fiber due to fiber loss. Hence, for long fibers ($L \gg L_{eff}$) the effective fiber length is defined as

$$L_{eff} = \frac{1-\exp(-\alpha L)}{\alpha} \approx \frac{1}{\alpha}.$$

Therefore, the FWM power can be expressed as

$$P_{FWM} = \frac{\eta'}{9}D^2\gamma^2 P_1 P_2 P_3 \exp(-\alpha L)\left[\frac{1-\exp(-\alpha L)^2}{\alpha^2}\right], \quad (7)$$

and the FWM efficiency is

$$\eta' = \frac{\alpha^2}{\alpha^2 + (\Delta\beta')^2}\left[1 + \frac{4\exp(-\alpha L)\sin^2(\Delta\beta' L/2)}{(1-\exp(-\alpha L))^2}\right], \quad (8)$$

where γ is the non-linear coefficient $2\pi n_2/\lambda A_{\rm eff}$; n_2 is the fiber non-linear refractive index $= 2.68 \times 10^{-20}\,{\rm m^2/W}$; P_1, P_2, P_3 are the input channel powers; $A_{\rm eff}$ is the effective area of fiber core; and $L_{\rm eff}$ is the effective fiber length.

3. Results and discussion

In our calculations, we assume the case of equal input channel power of the three input channels such that $P_1 = P_2 = P_3 = P_{\rm o}$. Referring to ITU: T recommendation G.653 [10]

D	Fiber chromatic dispersion = 0.5 ps/km nm
$\partial D/\partial\lambda$	Dispersion slope = 0.085 ps/km nm^2
$\partial^2 D/\partial\lambda^2$	Dispersion curvature = 2.3776×10^{-4} ps/km nm^3
$\partial^3 D/\partial\lambda^3$	Dispersion curvature = 7.0647×10^{-6} ps/km nm^4
α	Attenuation factor = 0.25 dB/km

In Fig. 1, we describe the effect of FWM crosstalk introduced in the optical fiber under the combined effect of 2OD, 3OD, 4OD and 5OD dispersion parameter with the phase matching factor, at $m = 0.63$, when three channels, each having equal channel power i.e. Po are transmitted through the fiber. For these calculations, the effective fiber core area is 50 μm^2, fiber loss 'α' = 0.25 dB/km and the fiber non-linear refractive index 'n_2' = $2.68 \times 10^{-}20\,{\rm m^2/W}$. The pump wavelengths are taken as 1558 and 1558.8 nm, resulting in the channel spacing of 0.8 nm. We plot FWM power versus channel power for different combinations cases of 2OD, 3OD, 4OD and 5OD dispersion parameters as shown in Figs. 1–6. The plot with the 2OD + 3OD + 4OD + 5OD term

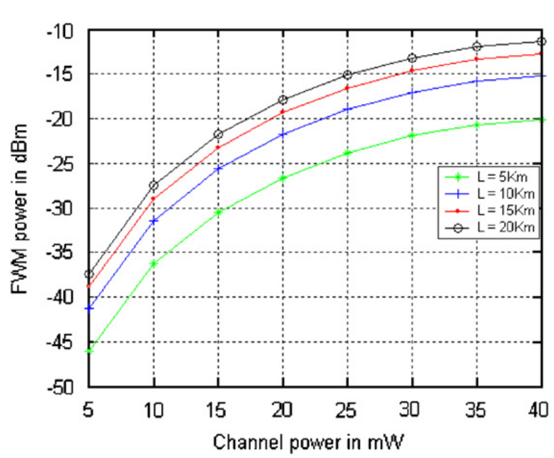

Fig. 1. FWM power versus input channel power at different values of distances under the combined effect of $\beta_2 + \beta_3 + \beta_4 + \beta_5$.

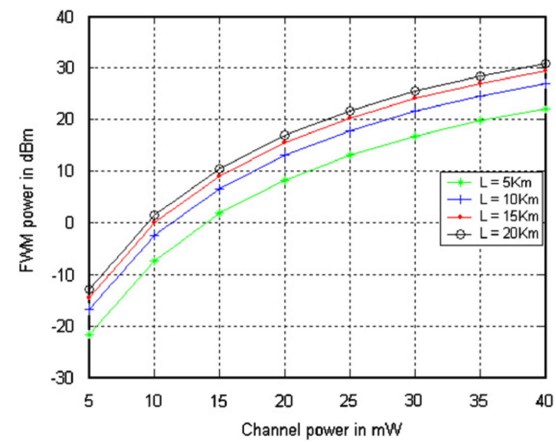

Fig. 2. FWM power versus input channel power at different values of distances under the combined effect of $\beta_3 + \beta_5$.

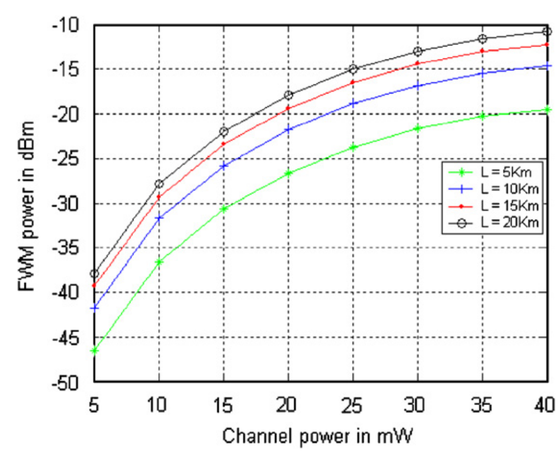

Fig. 3. FWM power versus input channel power at different values of distances under the combined effect of $\beta_2 + \beta_3$.

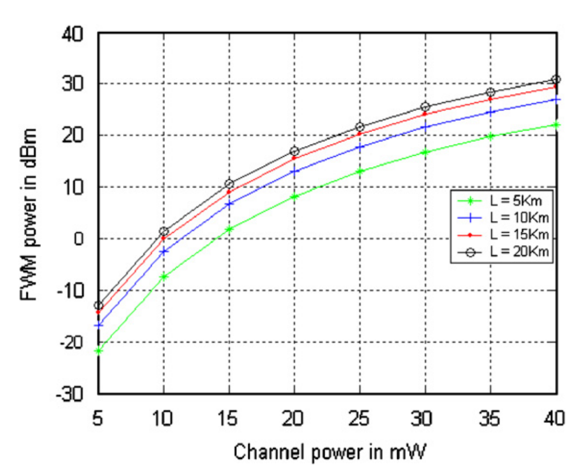

Fig. 4. FWM power versus input channel power at different values of distances under the combined effect of β_3.

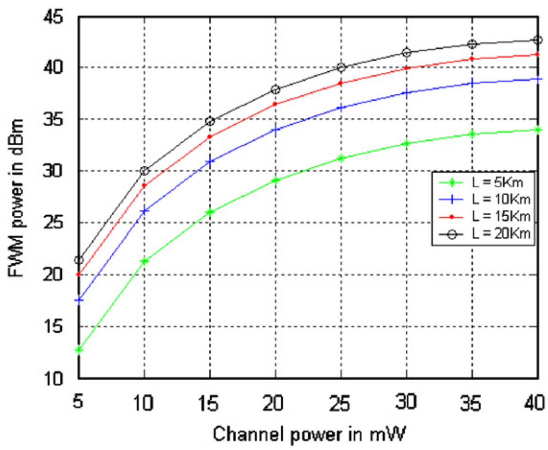

Fig. 5. FWM power versus input channel power at different values of distances under the combined effect of β_4.

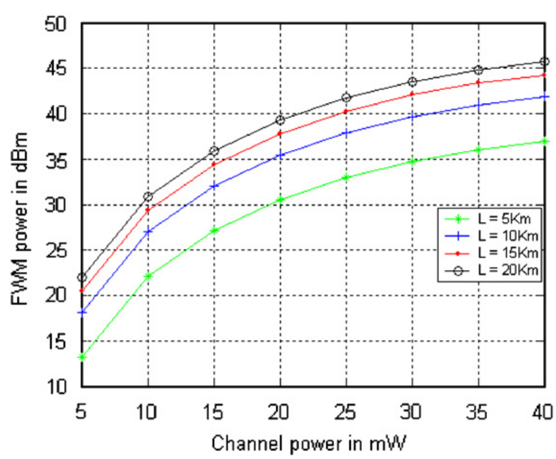

Fig. 6. FWM power versus input channel power at different values of distances under the effect of only β_5.

Table 1. Comparison of FWM power reduction due to different combinations for fiber length of 5 km

Dispersion parameter	FWM power in dBm when channel power = 5 mW	FWM power in dBm when channel power = 40 mW
3OD + 5OD only	−22	22
2OD + 3OD only	−47	−19
3OD only	−22	22
4OD only	14	34
5OD only	14	37
2OD + 3OD + 4OD + 5OD only	−47	−20

is shown in Fig. 1 and from this figure it is observed that the FWM power is −47 dBm at 5 mW channel power for 5 km length. Fig. 2 shows the combined effect of the 3OD + 5OD dispersion parameter on FWM power, and the variation from an earlier case, i.e.

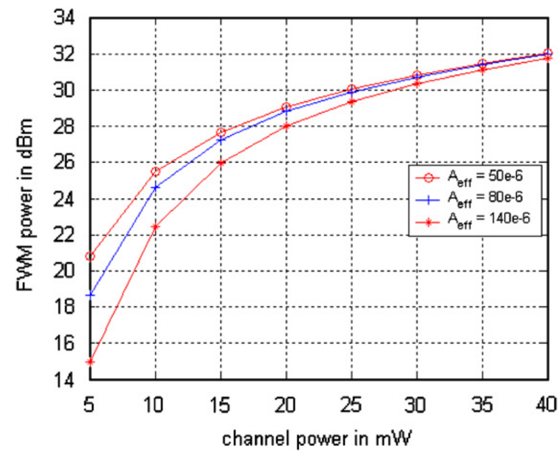

Fig. 7. FWM power versus input channel power at different values of core effective area under the effect of only β_5.

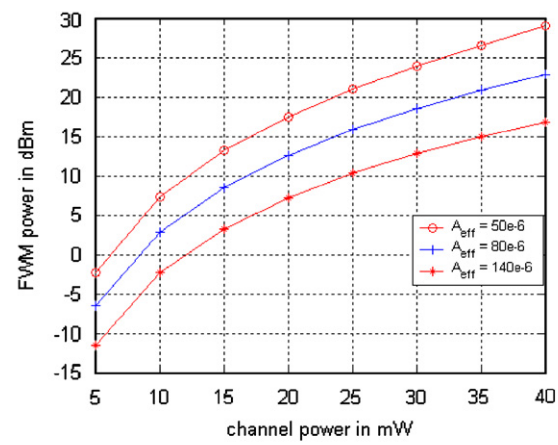

Fig. 8. FWM power versus input channel power at different values of core effective area under the effect of only β_4.

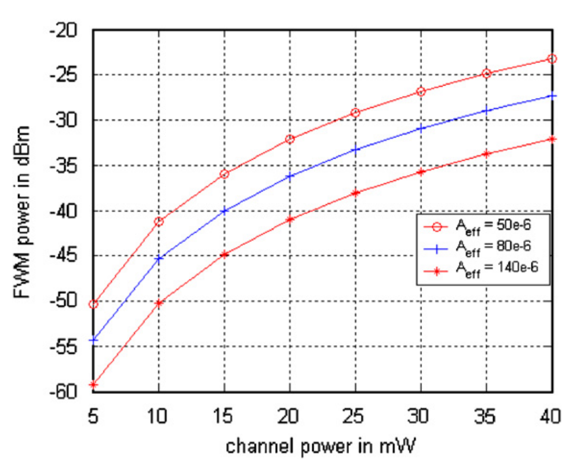

Fig. 9. FWM power versus input channel power at different values of core effective area under the combined effect of $\beta_2+\beta_3+\beta_4+\beta_5$.

Fig. 1, is observed. Similarly, the plots of 3OD + 5OD, 2OD + 3OD, 3OD and 5OD only are shown in Figs. 3–6, respectively. It is seen that the FWM power reduces by combined effect 2OD + 3OD + 4OD + 5OD dispersion terms.

The comparison of FWM power reduction due to different combinations is also given in Table 1.

From the above result, it is clear that the impact of 3OD, 4OD and 5OD is small as compared with 2OD but still contributes when the combined terms are considered.

Figs. 7–9 shows the impact of core effective area (A_{eff}) when the length of fiber is constant and 80 km and it is observed that as core effective increases FWM decreases.

4. Conclusion

This paper presents the detailed theoretical analysis of the influence of higher-order dispersion effect (2OD, 3OD, 4OD and 5OD) on FWM power. It is observed that the higher-order dispersion term has significant impact on FWM power. The impact decreases as the order of the dispersion term increases. The impact of 3OD, 4OD and 5OD is small as compared with 2OD but still contributes when the combined terms are considered. It is concluded that under the combined effect of second-, third-, fourth- and fifth-order dispersion parameters, the cross talk introduced by FWM reduces.

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