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Improved Small Signal Analysis for Dispersive Optical Fiber Communication Systems

Ajay K. Sharma*, R. K. Sinha**, Shatendra K. Sharmay+, Sandeep K. Arya*, R. S. Kalerz++

Summary

In this paper an improved small signal analysis for analyzing the influence of the higher-order dispersion on dispersive optical communication systems has been presented. A modified conversion matrix has been reported that gives the transfer function of intensity and phase from the fiber input to fiber output for any laser source including the influence of higher-order dispersion terms. In addition, analysis is applicable to evaluate the impact of higher-order dispersion on small signal frequency response and relative intensity noise (RIN) of a ultrafast laser diode.

1 Introduction

The invention of the erbium-doped fiber amplifier (EDFA) [1, 2] paved the way for the development of high bit rate, all optical ultra long-distance communication systems. Specifically, periodic compensation of fiber loss by EDFA's eliminates the need for electronic repeaters along the transmission line and enables the construction of all-optical communication systems in which the transmission distance is limited by the fiber chromatic dispersion rather by the fiber loss [3] because it introduces signal distortion and noise [4–8]. However, if conventional 1.3 μm zero dispersion optical fiber systems and networks are used for the 1.55 μm signal light, they exhibit a significant dispersion yielding, e. g. limitations with respect to transmission bandwidth [9–10].

Wang et al. [11] developed a new approach to investigate the influence of the dispersion on optical fiber communication systems using small signal analysis. A conversion matrix describing the transfer function of intensity and frequency modulation at fiber input to the intensity and frequency modulation at fiber output was reported and the results were obtained to analyze the perfomance of optical communication systems. A theory describing the propagation of signal and noise through a lossless linear dispersive single mode fiber zero first-order dispersion point was presented, recently, by Crognale [12]. Recalling the small signal approach reported in [11], a simple and exhaustive treatment was developed to study the small signal and noise transmission characteristics in the frequency domain of a high performance laser diode together with a linear dispersive fiber. The impact of second-order dispersion term on the modulation and noise properties of an ultrafast laser diode was obtained to study the total frequency response and the relative intensity noise (RIN) at output of the linear single-mode fiber but the impact of other higher-order dispersion terms were neglected.

In this paper, we have extended the work reported in [11, 12] by presenting an improved analysis for analyizing the influence of the higher-order dispersion on dispersive optical communication system. A modified conversion matrix has been reported that gives the transfer function of intensity and phase from the fiber input to the fiber output for any laser source including the influence of higher-order dispersion terms. Moreover, this theory is applicable to evaluate the impact of higher-order dispersion on the small signal frequency response and RIN of a ultrafast laser diode similarly as mentioned in [12]. In addition a generalized conversion matrix is presented to analyze the small signal and noise transmission characteristics of ultrafast laser diode through a dispersive medium by incorporating the influence of any order of dispersion term.

We will first in section II present the modified analysis for a phase and intensity modulated signal propagating through a dispersive medium, yielding the desired conversion matrix inclusive influence of higher-order dispersion terms. Section III discusses the application of conversion matrix to the small signal intensity and frequency modulation characteristics of laser diode and finally RIN at output of a dispersive fiber including the impact of higher-order dispersion.

Address of authors:

* Department of Electronics and Communication Engineering
Regional Engineering College
Jalandhar 144011, Punjab, India

** Department of Applied Sciences
Delhi College of Engineering
Bawana Road, Delhi-42

\+ University Science Instrumentation Center (USIC)
Jawaharlal Nehru University
New Delhi-110070, India

++ Department of Electronics and Communication Engineering
Sant Longowal Institute of Engineering and Technology
Longowal-148106, India.

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2 Analysis of intensity and phase of an optical field at the fiber output

In this section we will present an improved transfer function using similar appproach reported in [11, 12] for a linear dispersive single-mode fiber, including the effects of the higher-order dispersion terms i. e. second-, third- and fourth-order dispersion (2OD, 3OD and 4OD) terms. In fact, the chromatic dispersion is the most significant limiting factor to degrade the performance of ultrafast long-distance broadband optical communication systems. Therefore, it is important not only to analyze the impact of second-order dispersion, but also its slope (third-order dispersion, 3OD) and curvature (fourth-order dispersion, 4OD) [13] for ultra fast long-distance broadband optical communication systems.

Let the electric field at the input of fiber from a single mode laser diode [14].

$$E(t) = E_{in}(t)e^{j\omega_0 t} \tag{1}$$

Where ω_0 is the mean circular optical emission frequency and $E_{in}(t)$ is given by [11]

$$E_{in}(t) = \sqrt{S_{in}(t)}e^{j\phi_{in}(t)} \tag{2}$$

is the slowly varing field amplitude, $S_{in}(t)$ and $\phi_{in}(t)$ are, respectively, the input photon density and input phase. As in [15], the propagation of the signal through an optical fiber can be described by propagation term $e^{-j\beta L}$ with length L of the transmission fiber and the phase constant β (losses are neglected) which may be expanded around $\omega = \omega_0$ as mentioned in [15–18].

$$\beta = \beta_0 + (\omega-\omega_0)\frac{\partial\beta}{\partial\omega} + \frac{1}{2}(\omega-\omega_0)^2\frac{\partial^2\beta}{\partial\omega^2} + \frac{1}{6}(\omega-\omega_0)^3\frac{\partial^3\beta}{\partial\omega^3} + \frac{1}{24}(\omega-\omega_0)^4\frac{\partial^4\beta}{\partial\omega^4} + \dots \tag{3}$$

where $\partial\beta/\partial\omega = \tau$ is group-delay for unit length [19].

$$\frac{\partial^2\beta}{\partial\omega^2} = \frac{1}{2}\frac{\lambda^3}{2(2\pi c)^2}\left[\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 2\frac{\partial\beta}{\partial\lambda}\right] \tag{4}$$

is second-order dispersion,

$$\frac{\partial^3\beta}{\partial\omega^3} = \frac{1}{6}\frac{\lambda^4}{(2\pi c)^3}\left[\lambda^2\frac{\partial^3\beta}{\partial\lambda^3} + 6\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 6\frac{\partial\beta}{\partial\lambda}\right] \tag{5}$$

is third-order dispersion and

$$\frac{\partial^4\beta}{\partial\omega^4} = \frac{1}{24}\frac{\lambda^5}{(2\pi c)^4}\left[\lambda^3\frac{\partial^4\beta}{\partial\lambda^4} + 12\lambda^2\frac{\partial^3\beta}{\partial\lambda^3} + 36\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 24\frac{\partial\beta}{\partial\lambda}\right] \tag{6}$$

is the fourth-order dispersion (in (4), (5) and (6), λ is the wavelength and c is the vacuum velocity of light). Then, recalling (3) the following expression is obtained for propagation term $e^{-j\beta L}$:

$$e^{-j\beta L} = e^{-j\beta_0 L - jL(\omega-\omega_0)\frac{\partial\beta}{\partial\omega} - jL\frac{1}{2}(\omega-\omega_0)^2\frac{\partial^2\beta}{\partial\omega^2} - jL\frac{1}{6}(\omega-\omega_0)^3\frac{\partial^3\beta}{\partial\omega^3} - jL\frac{1}{24}(\omega-\omega_0)^4\frac{\partial^4\beta}{\partial\omega^4}} \tag{7}$$

If the propagation term $e^{-j\beta L}$ with expanded fi is applied to the the Fourier transform of the input optical signal as mentioned in [11], we obtain the complex field amplitude at the output of the fiber in the frequency domain

$$E_{out}(j\omega) = E_{in}(j\omega)e^{-j\beta L} \tag{8}$$

or time domain according to

$$E_{out}(t) = e^{j(\omega_0 t - \beta_0 L)}\left[e^{\frac{1}{2}L\frac{\partial^2\beta}{\partial\omega^2}\frac{\partial^2}{\partial t^2} + \frac{L}{6}\frac{\partial^3\beta}{\partial t^3} - \frac{j}{24}L\frac{\partial^4\beta}{\partial\omega^4}\frac{\partial^4}{\partial t^4}}\right] E_{in}(t-\tau L). \tag{9}$$

As reported in [11, 12, 14–18], we have neglected the phase and group delay $\beta_0 L$ and $L(\partial\beta/\partial\omega)$ or τL because both terms produce only phase delay of the carrier signal and time delay of the modulation signal and have no influence on the distortion of the signal and define the following dispersion parameters:

$$F_2 = \frac{L}{2}\frac{\lambda^3}{2(2\pi c)^2}\left[\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 2\frac{\partial\beta}{\partial\lambda}\right] \tag{10}$$

$$F_3 = -\frac{L}{6}\frac{\lambda^4}{(2\pi c)^3}\left[\lambda^2\frac{\partial^3\beta}{\partial\lambda^3} + 6\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 6\frac{\partial\beta}{\partial\lambda}\right] \tag{11}$$

$$F_4 = \frac{L}{24}\frac{\lambda^5}{(2\pi c)^4}\left[\lambda^3\frac{\partial^4\beta}{\partial\lambda^4} + 12\lambda^2\frac{\partial^3\beta}{\partial\lambda^3} + 36\lambda\frac{\partial^2\beta}{\partial\lambda^2} + 24\frac{\partial\beta}{\partial\lambda}\right]. \tag{12}$$

The field amplitude at the fiber output (9) can be written as

$$E_{out}(t) = e^{j\omega_0 t}\sqrt{S_{out}(t)}e^{j\phi_{out}(t)} \tag{13}$$

$$= e^{j\omega_0 t}\left(e^{jF_2\frac{\partial^2}{\partial t^2} - F_3\frac{\partial^3}{\partial t^3} - jF_4\frac{\partial^4}{\partial t^4}}\right)\left(\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\right) \tag{14}$$

$$= E_{in}(t) + \Delta E(t) \tag{15}$$

where $S_{out}(t)$ and $\phi_{out}(t)$ are, respectively, the intensity and phase of the field amplitude at fiber output and $\Delta E(t)$ is expressed as

$$\Delta E(t) = E_{out} - E_{in}(t) \tag{16}$$

$$= e^{j\omega_0 t}\left(e^{jF_2\frac{\partial^2}{\partial t^2} - F_3\frac{\partial^3}{\partial t^3} - jF_4\frac{\partial^4}{\partial t^4}} - 1\right)\left(\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\right) \tag{17}$$

in which $\Delta E(t)$ is assumed to be small

$$|\Delta E(t)| \ll |E_{in}(t)|. \tag{18}$$

From (17), with the help of approximation (18), we can obtain the expressions of intensity at output of the fiber [11, 12] as

$$S_{out}(t) = |E_{in}(t) + \Delta E(t)|^2 \approx |E_{in}(t)|^2 + 2 \cdot \Re[E^*_{in}(t) \cdot \Delta E(t)]$$
$$= S_{in}(t) + 2\Re\Big[\sqrt{S_{in}(t)} e^{-j\phi_{in}(t)}$$
$$\cdot\Big(e^{jF_2 \frac{\partial^2}{\partial t^2} - F_3 \frac{\partial^3}{\partial t^3} - jF_4 \frac{\partial^4}{\partial t^4}} - 1\Big)\sqrt{S_{in}(t)} e^{j\phi_{in}(t)}\Big] \tag{19}$$

and similarly as

$$\phi_{out}(t) = \phi_{in}(t) + \Im\left[\frac{\Delta E(t)}{E_{in}(t)}\right] = \phi_{in}(t) + \Im\left(\frac{\left[\left(e^{jF_2 \frac{\partial^2}{\partial t^2} - F_3 \frac{\partial^3}{\partial t^3} - jF_4 \frac{\partial^4}{\partial t^4}} - 1\right)\right]\sqrt{S_{in}(t)} e^{j\phi_{in}(t)}}{\sqrt{S_{in}(t)} e^{j\phi_{in}(t)}}\right). \tag{20}$$

(19) and (20) derived above are the general equations describing the intensity and phase of an optical field after propagating through a dispersive optical fiber. These equations are valid for arbitrary input intensity and phase as long as dispersion induced field amplitude $\Delta E(t)$ is assumed to be small compared to the input field $E_{in}(t)$. Further, (19) and (20) can be simplified using small signal analysis.

The small signal analysis implies that the average intensity $\langle S \rangle$ is larger than the noise or modulation term $\Delta S_{in}(t)$

$$S_{in}(t) = \langle S \rangle + \Delta S_{in}(t) \tag{21}$$

with satisfying following relation:

$$\langle S \rangle \gg \Delta S_{in}(t). \tag{22}$$

As reported in [11, 12] in the small signal approach, the field amplitude $\sqrt{S_{in}(t)}$ can be linearized as

$$\sqrt{S_{in}(t)} \approx \sqrt{\langle S \rangle}\left(1 + \frac{\Delta S_{in}(t)}{2\langle S \rangle}\right). \tag{23}$$

After neglecting the product of small signal [11, 12, 20], we can introduce the following approximations:

$$\frac{\partial^n}{\partial t^n}\left(\sqrt{S_{in}(t)} e^{j\phi_{in}(t)}\right) \approx \sqrt{\langle S \rangle} e^{j\phi_{in}(t)} \frac{\partial^n}{\partial t^n} \cdot \left[\frac{\Delta S_{in}(t)}{2\langle S \rangle} + j\phi_{in}(t)\right]. \tag{24}$$

Inserting (24) into (19) and (20), we obtain

$$S_{out}(t) = \langle S \rangle + 2\Re\Big[\langle S \rangle \cdot e^{j\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right) - F_3 \frac{\partial^3}{\partial t^3}}$$
$$\left(\frac{\Delta S_{in}(t)}{2\langle S \rangle} + j\phi_{in}(t)\right)\Big] \tag{25}$$

and similarly as

$$\phi_{out}(t) = \Im\left[e^{j\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right) - F_3 \frac{\partial^3}{\partial t^3}}\left(\frac{\Delta S_{in}(t)}{2\langle S \rangle} + j\phi_{in}(t)\right)\right]. \tag{26}$$

If F_3 is small, the exponential operator can be written as [11, 12]

$$e^{j\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right) - F_3 \frac{\partial^3}{\partial t^3}} \approx e^{j\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)} \cdot \left(1 - F_3 \frac{\partial^3}{\partial t^3}\right). \tag{27}$$

Further, the operator $e^{j(F_2(\partial^2/\partial t^2) - F_4(\partial^4/\partial t^4))}$ can be split into its real and imaginary parts and (27) becomes

$$\left[\cos\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right) + j\sin\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)\right]$$
$$\left(1 - F_3 \frac{\partial^3}{\partial t^3}\right). \tag{28}$$

In this way the relations for intensity and phase derived at the fiber output are:

$$\Delta S_{out}(t) = \cos\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)\left[\Delta S_{in}(t) - F_3 \Delta \dddot{S}_{in}(t)\right]$$
$$-2\langle S \rangle \sin\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)\left[\phi_{in}(t) - F_3 \dddot{\phi}_{in}(t)\right] \tag{29}$$

and

$$\phi_{out}(t) = \cos\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)\left[\phi_{in}(t) - F_3 \dddot{\phi}_{in}(t)\right]$$
$$+\sin\left(F_2 \frac{\partial^2}{\partial t^2} - F_4 \frac{\partial^4}{\partial t^4}\right)\left[\frac{\Delta S_{in}(t)}{2\langle S \rangle} + F_3 \frac{\Delta \dddot{S}_{in}(t)}{2\langle S \rangle}\right] \tag{30}$$

with the output modulation or noise given by

$$\Delta S_{out}(t) = S_{out}(t) - \langle S \rangle \tag{31}$$

and dots denote the time derivatives. Recalling the relation between frequency and phase, as [11, 12] results may be expressed with modified conversion matrix that describes the relation between intensity and frequency, including the second-, third-, and fourth-order dispersion terms as shown in (32) and is the extension of the expression reported in [11, 12].

$$\begin{pmatrix} \Delta S_{out}(\omega) \\ \dot{\phi}_{out}(\omega) \end{pmatrix} = \begin{pmatrix} \cos(\omega^2 F_2 + \omega^4 F_4)[1 + j\omega^3 F_3] & 2\langle S \rangle \sin(\omega^2 F_2 + \omega^4 F_4)\left[\omega^2 F_3 - \frac{j}{\omega}\right] \\ \frac{\sin(\omega^2 F_2 + \omega^4 F_4)[\omega^4 F_3 - j\omega]}{2\langle S \rangle} & \cos(\omega^2 F_2 + \omega^4 F_4)[1 + j\omega^3 F_3] \end{pmatrix}\begin{pmatrix} \Delta S_{in}(\omega) \\ \dot{\phi}_{in}(\omega) \end{pmatrix}. \tag{32}$$

The modified conversion matrix (32) derived above may be generalized for other higher-order dispersion terms. The generalized form of the conversion matrix may be written as under and is valid to derive conversion matrixes reported in [11, 12] and in this paper after excluding or including the respective higher- order dispersion terms.

$$\begin{pmatrix} \Delta S_{out}(\omega) \\ \phi_{out}(\omega) \end{pmatrix} = \begin{pmatrix} \cos\left(\omega^2 F_2 + \ldots + \omega^n F_n\right)\left[1 + j\omega^3 F_3 \ldots + j\omega^m F_m\right] & -\frac{j2\langle S\rangle}{\omega}\sin\left(\omega^2 F_2 + \ldots + \omega^n F_n\right)\left[1 + j\omega^3 F_3 \ldots + j\omega^m F_m\right] \\ -\frac{j\omega \sin\left(\omega^2 F_2 + \ldots \omega^n F_n\right)\left[1 + j\omega^3 F_3 \ldots + j\omega^m F_m\right]}{2\langle S\rangle} & \cos\left(\omega^2 F_2 + \ldots \omega^n F_n\right)\left[1 + j\omega^3 F_3 \ldots + j\omega^m F_m\right] \end{pmatrix} \begin{pmatrix} \Delta S_{in}(\omega) \\ \phi_{in}(\omega) \end{pmatrix} \quad (33)$$

where n = 2; 4; 6; ...; and m = 1; 3; 5; ... If we assume that only intensity modulation is present and there is no phase modulation, we can obtain transfer function $\cos(\omega^2 F_2 + \omega^4 F_4)[1 + j\omega^3 F_3]$ in comparison with $\cos(\omega^2 F_2)$ [11] obtained exclusive third- and forth-order dispersion terms.

On the other hand, if we assume that only phase modulation is present, the intensity modulation $S_{out}(\omega)$ at output of a dispersive fiber due to FM-AM conversion can be expressed as

$$\Delta S_{out}(\omega) = 2\langle S\rangle \sin\left(\omega^2 F_2 + \omega^4 F_4\right)\left[\omega^2 F_3 - \frac{j}{\omega}\right]. \quad (34)$$

Using above analysis it is easy to calculate the ??? in the presence of either only intensity modulation or phase modulation through a linear dispersive fiber including the influence of higher-order dispersion terms. In addition, (34) dosn't need the calculation of Bessel function.

3 Small signal intensity and frequency modulation characteristics and RIN at output of a dispersive single-mode fiber

If we consider a modulation of the injection current around the mean value $\langle I\rangle$, the small-signal response of the intrinsic laser may be obtained with small-signal modulation current ΔI, so that $|\Delta I| \ll \langle I\rangle$, yielding, in frequency domain

$$\Delta S_{in}(\omega) = \left(\frac{\tau_{ph}}{e}\right) H(\omega)\Delta I(\omega) \quad (35)$$

where τ_{ph} is the photon lifetime, e is the elementary charge and the small-signal modulation transfer function

$$H(\omega) = \frac{\omega_r^2}{(j\omega)^2 + j\omega\Gamma + \omega_r^2}. \quad (36)$$

Here, the Γ is the damping rate. The relation between frequency modulation and intensity modulation (chirp) may be described by [14, 21]

$$\phi_{in}(\omega) = \frac{\alpha}{2}\left(j\omega + \omega_g\right)\frac{\Delta S_{in}(\omega)}{\langle S\rangle} \quad (37)$$

where, α is the linewidth enhancement factor, and ω_g is the device-specific characteristic frequency (if the chirp characteristics are mainly due to nonlinear gain, we have $\omega_g \approx \Gamma$). The intensity modulation at the fiber output can be obtained by inserting above (35), (36) and (37) into (32)

$$\Delta S_{out}(\omega) = \left(\frac{\tau_{ph}}{e}\right)\left(1 + j\omega^3 F_3\right) \left[\cos\left(\omega^2 F_2 + \omega^4 F_4\right) - j\alpha \sin\left(\omega^2 F_2 + \omega^4 F_4\right) \frac{j\omega + \omega_g}{\omega}\right] \cdot H(\omega) \cdot \Delta I(\omega). \quad (38)$$

(38) gives intensity modulation at output of a dispersive fiber including the effects of 2OD, 3OD and 4OD terms. Recalling the results reported in [11, 12], and from (38) we can derive the following relation between small signal response $[|\Delta S_{out}(\omega)|/|\Delta I(\omega)|]_{F_2,F_4=0}$ neglecting the second- and fourth-order dispersion terms or F_1 in [12] and the small signal frequency response $[|\Delta S_{out}(\omega)|/|\Delta I(\omega)|]_{F_2=0}$, calculated including this term of higher-order (F_4).

$$\left[\frac{|\Delta S_{out}(\omega)|}{|\Delta I(\omega)|}\right]_{F_2=0} = \left(\left[\cos\left(\omega^4 F_4\right) + \alpha \sin\left(\omega^4 F_4\right)\right]^2 + \left(\frac{\alpha\omega_g}{\omega}\right)^2 \sin^2\left(\omega^4 F_4\right)\right) \cdot \left[\frac{|\Delta S_{out}(\omega)|}{|\Delta I(\omega)|}\right]_{F_2,F_4=0}. \quad (39)$$

(39) shows that at high modulation frequencies the fourth-order dispersion term introduces an enhancement factor $([\cos(\omega^4 F_4) + \alpha \sin(w^4 F_4)]^2 + (\alpha\omega_g/\omega)^2 \sin^2(\omega^4 F_4))$ of the transfer function if $F_2 = 0$.

Using (38) we can derive the expressions for intensity modulation at the output of the fiber $\Delta S_{out}(\omega)$ reported in [11] and [12] if F_3, $F_4 = 0$ and $F_4 = 0$ respectively. Similarly, we can derive and analyze the impact of other independent and combined higher-order dispersion terms on intensity modulation if $F_2 = 0$, $F_3 = 0$, F_2, $F_3 = 0$, and F_2; $F_4 = 0$. In addition, this small signal analysis may be applied to evaluate the chromatic dispersion on the small signal frequency response and RIN of a ultrafast laser diode similarly as reported in [11, 12] in order to see the influence of higher-order dispersion terms all together and independently.

It is well known that RIN is significantly enhanced after passing through a dispersive fiber because of different delays of the spectral components within the spectral width of the laser and due to higher-order dispersion terms it may not be neglected for very long broadband transmission links.

The FM-AM noise conversion without considering the intensity noise of the laser source has been investigated in [14, 22]

Recalling the results obtained in [11, 12], and the modified conversion matrix (32), one finally obtains the RIN at the fiber output due to higher-order dispersion terms i.e. at F_3 and F_4, neglecting F_2 for simplification.

$$\left[\frac{\mathrm{RIN}}{\Delta f}\right]_{F_2=0} = \left(\left[\cos\left(\omega^4 F_4\right)+\alpha\sin\left(\omega^4 F_4\right)\right]^2+\left(\frac{\alpha\omega_g}{\omega}\right)^2\sin^2\left(\omega^4 F_4\right)\right)\cdot \left[\frac{\mathrm{RIN}}{\Delta f}\right]_{F_2 m F_4=0}. \tag{40}$$

(40) show that the product $[\mathrm{RIN}/\Delta f]_{F_2,F_4=0}$ depends only on third-order dispersion as reported in [12], and the enhancement factor $([\cos(\omega^4F_4)+\alpha\sin(w^4F_4)]^2+(\alpha\omega_g/\omega)^2 \sin^2(\omega^4F_4))$ depends on F_3 F_4.

Results reported in [11] (see (33) and (40)) and [12] (see (47) and (52)) for intensity modulation and RIN at the fiber output may also be derived using modified conversion matrix to analyze the impact of higher-order dispersion terms. The modified conversion matrix reported in this paper permits to derive, for any arbitrary frequency and intensity modulation (or noise) at the fiber input, the corresponding frequency and intensity modulation (or noise) at the fiber output, taking also into account the correlation between the phase and intensity modulation and considering the higher-order dispersion effects (up to fourth-order dispersion).

4 Conclusion

In this paper we have developed the theory to investigate the influence of higher-order dispersion terms on optical communication systems using small signal analysis. The theory developed in this paper may be considered as extension of the results reported in [11, 12], where the impact of higher-order dispersion terms were neglected. Modified conversion matrix [see (32)] obtained in this paper permit immediately to evaluate, with analytical closed-form expressions, the impact of 2OD, 3OD and 4OD terms independently or combined on the modulation and noise properties of a linear single mode fiber transmission system. The transfer function of intensity and phase from the fiber input to fiber output for every source emission wavelength inclusive the impact of higher-order dispersion terms may be obtained using modified conversion matrix. Further, it has been investigated that the higher-order dispersion terms introduces an enhancement factor of transfer function and RIN characteristics at very high modulation frequency. Results have been presented to obtain the small signal frequency response of phase and intensity modulation of a laser diode together in a dispersive fiber. In addition a generalized conversion matrix is presented to analyze the small signal and noise transmission characteristics of ultrafast laser diode through a dispersive medium by incorporating the influence of any order of dispersion term.

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