Femto-Second Transform Limited Pulse Generation with Higher Order Dispersion Effects for Dispersion-Shifted Optical Communication System

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The paper deals with the influence of higher-order effects of dispersion on the femto-second transform limited pulse generation by compensating for linear chirp of self-phase modulation spectra in the dispersion-shifted fibers. It has been shown that the minimum propagation length with first-order dispersion term is 23 m, as reported earlier. If the higher-order dispersion effects are taken into consideration, this length is reduced to 11.5 m. With compensation of the first-order dispersion term, this length can be enhanced to $6.8161 \times 10^3$ km. This length can further be improved to $6.0343 \times 10^3$ km by compensation of first- and second-order dispersion terms together. The minimum pulse width and linewidth product without dispersion, with dispersion including higher-order dispersion effects, and with dispersion compensation, is found to be 0.44, 0.4418, and 0.4411, respectively.

Keywords optical time division multiplexing, self-phase modulation, dispersion, higher-order dispersion, Gaussian approximation, nonlinearity

Introduction

It is of utmost importance to calculate and compensate the dispersion and nonlinear effects in optical communication systems because these are the physical limitations for high-speed transmission systems resulting from the transmission properties of optical fiber. With the current trends toward ultrahigh bit rate transmission systems, the effect of first-, second-, and higher-order dispersion is becoming increasingly significant [1]. As a high bit rate optical transmission system is based on multi/demultiplexing in which
the information carrier is a continuous sequence of ultra-short pulses (with a repetition rate in the giga-Hertz range) generated by semiconductor laser diodes, this requires direct modulation of a DFB (distributed feedback) laser with the electrical pulses having a suitable duration to excite only the first peak of the relaxation oscillations. The emitted pulses exhibit a strong chirp that distorts the phase and broadens the corresponding intensity spectrum by using a Gaussian profile and first-order approximation of a pump pulse envelope. The analytical description was obtained, and a good approximation with experimental results was found [2]. Theoretical and experimental work on pulse compression based on linear chirp compensation of self-phase modulation (SPM) in dispersion-shifted (DS) fibers with pulse duration of 233 fs and DOP of 0.4 at a pump peak power of 19.8 W was performed, and good agreement between computed and experimental results was found with approximation of the Gaussian pulse envelope [3]. The GVD (group velocity dispersion) of V-groove semiconductor lasers (for femto-second optical pulses) was measured at a wavelength near 1.55 μm [4]. The numerical investigation of the interaction of ultrashort broadband optical pulses with narrow fiber grating was performed [5].

A technique for obtaining ultrashort transform limited (TL) pulses by super-continuum radiation used in optical time division multiplexing (OTDM) or wavelength division multiplexing (WDM) is discussed and by using a parabolic law that describes the behavior of the square pulse width (full width at half maximum: FWHM) at the output of the DS fiber with pump peak intensity at the input for a fixed fiber length with single-mode fiber [3].

In this paper we investigate the influence of higher-order dispersion nonlinearly on the minimum fiber length and minimum pulse width and linewidth product (Δtmin · Δνmin). In the next section we have derived the expression for the minimum fiber length and for the minimum pulse width and linewidth product in the presence of higher-order dispersion and nonlinear effects. In the third section the results of dispersion compensation for minimum fiber length and minimum pulse width and linewidth product are presented, and in the last section we draw conclusions.

Theory

Self-phase modulation (SPM) is a phenomenon by virtue of which a traveling intense and narrow optical pulse changes its phase because of a change in the refractive index that causes a frequency sweep inside the pulse envelope. The SPM mechanism is described using Maxwell equations with a nonlinear polarization term. The slowly varying pulses can be described by the following differential equation [3]:

\[
\frac{\partial A}{\partial z} + \frac{1}{V_g} \frac{\partial A}{\partial t} = \frac{j\omega_o n_2}{2c} |A|^2 A,
\]

where \( A \) is the electric field envelope, \( V_g \) is group velocity, \( \omega_o \) is the frequency of the pump source, \( n_2 \) is the nonlinear refractive index, and \( c \) is the velocity of light.

If we have \( a(t, z) \) and \( \alpha(t, z) \) as the amplitude and phase of the electrical field, then the electric field envelope can be written as

\[
A(t, z) = a(t, z)e^{j\alpha(t, z)}.
\]

Now substituting equation (2) into equation (1) leads to

\[
\frac{\partial a(t, z)}{\partial z} + \frac{1}{V_g} \frac{\partial a(t, z)}{\partial t} + j a(t, z) \left[ \frac{\partial \alpha(t, z)}{\partial z} + \frac{1}{V_g} \frac{\partial \alpha(t, z)}{\partial t} \right] = \frac{j\omega_o n_2}{2c} |a|^2 a(t, z).
\]
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Equating real and imaginary parts on both sides of equation (3) we get

\[ \frac{\partial a(t, z)}{\partial z} + \frac{1}{V_g} \frac{\partial a(t, z)}{\partial t} = 0, \]  
(4)

\[ \frac{\partial \alpha(t, z)}{\partial z} + \frac{1}{V_g} \frac{\partial \alpha(t, z)}{\partial t} = \frac{\omega_0 n_2}{2c} a^2. \]  
(5)

We get the solution of equations (4) and (5) as

\[ a(\tau) = a_o e^{\left( t - \frac{z}{V_g} \right)}, \]  
(6)

\[ \alpha(t, z) = \frac{\omega_0 n_2}{2c} a_o^2 F^2(z), \]  
(7)

where \( F \) denotes function. Now using equations (6) and (7) with equation (2) we get

\[ A(t, z) = a_o e^{\left( t - \frac{z}{V_g} \right)} e^{j \omega_0 \frac{\omega_0 n_2}{2c} a_o^2 F^2(z)}. \]  
(8)

If \( \delta \omega(\tau) \) is local and instantaneous frequency, as given by [3] we have

\[ \delta \omega(\tau) = - \frac{\partial \alpha(t, z)}{\partial \tau} = \frac{\omega_0 n_2}{2c} \frac{\partial F^2(\tau)}{\partial \tau}. \]  
(9)

Now as the pulse envelope is of Gaussian profile and considering a first-order approximation,

\[ F^2(\tau) = e^{-\frac{\tau^2}{2\tau^2}} \quad \text{and} \quad F(\tau) = e^{-\frac{\tau^2}{2\tau^2}}. \]  
(10)

Substituting equation (10) in equation (8), the latter can be written as

\[ A(t, z) = a_o \left[ e^{\frac{-1}{2\tau^2}} - \frac{2\omega_0 n_2 P}{2} \frac{1}{\frac{1}{16\pi^2}} \right]. \]  
(11)

The approximate form of equation (11) is usually employed for calculating the linear chirp coefficients of Gaussian pulse in the time domain. As explained in [3] we have

\[ A(\Omega) = \frac{\tau_o a_o}{\sqrt{2\pi}} e^{j \frac{\gamma z P}{2}} \left[ e^{\frac{-s^2}{2}} \left( \frac{1 - \frac{1}{4\pi^2}}{1 + \frac{1}{4\pi^2}} \right) \right]. \]  
(12)

\[ \gamma = \frac{\omega_0 n_2}{2c}, \]  
(13)

where \( \Omega = \omega - \omega_o \), \( \gamma \) is the nonlinearity coefficient of the fiber, and \( P \) is the power.

Now the phase of \( \Phi(\Omega) \) can be written as [3]

\[ \Phi(\Omega) = \gamma z P - \frac{1}{2} \arctan(2\gamma z P) + \frac{\tau_o^2 \gamma z P}{1 + 4\gamma^2 z^2 P^2} \Omega^2. \]  
(14)

The Fourier transform of the propagating pulse envelope with fiber length \( L \) can be written as

\[ A(\Omega, L) = A(\Omega) e^{j(\Phi(\Omega) + \beta(\Omega)L)} e^{j \gamma z P}. \]  

where $\Phi(\Omega)$ can be written as [3]

$$
\Phi(\Omega) = C_{\omega} + \frac{1}{2} C_2 \Omega^2; \quad \Phi'(\Omega) = C_2 \Omega.
$$

and $C_2$ is the correlation coefficient. The propagation constant $\beta$ in terms of the Taylor series can be expanded around $\omega = \omega_o$ as mentioned in [6–9] as

$$
\beta(\omega) = \beta_o + (\omega - \omega_o) \frac{d\beta}{d\omega} + \frac{1}{2} (\omega - \omega_o)^2 \frac{d^2 \beta}{d\omega^2} + \frac{1}{6} (\omega - \omega_o)^3 \frac{d^3 \beta}{d\omega^3}
$$

$$
+ \frac{1}{24} (\omega - \omega_o)^4 \frac{d^4 \beta}{d\omega^4} + \cdots,
$$

(16)

where $\frac{d\beta}{d\omega} = \tau$ is the group delay for unit fiber length. Here

$$
\beta_2 = \frac{d^2 \beta}{d\omega^2} = -\frac{\lambda^2}{2\pi c} \frac{\partial \tau}{\partial \lambda}
$$

(17)

is first-order dispersion,

$$
\beta_3 = \frac{d^3 \beta}{d\omega^3} = \frac{\lambda^2}{(2\pi c)^2} \left[ \frac{\lambda^2}{\lambda^2} \frac{\partial^2 \tau}{\partial \lambda^2} + 2\lambda \frac{\partial \tau}{\partial \lambda} \right]
$$

(18)

is second-order dispersion, and

$$
\beta_4 = \frac{d^4 \beta}{d\omega^4} = \frac{\lambda^3}{(2\pi c)^3} \left[ \lambda^3 \frac{\partial^3 \tau}{\partial \lambda^3} + 6\lambda^2 \frac{\partial^2 \tau}{\partial \lambda^2} + 6\lambda \frac{\partial \tau}{\partial \lambda} \right]
$$

(19)

is third-order dispersion.

Now by differentiating equation (16) with respect to $\omega$ we have

$$
\beta'(\omega) = \beta_1 + 2(\omega - \omega_o)\beta_2 + (\omega - \omega_o)^2 \beta_3 + \frac{1}{3} (\omega - \omega_o)^3 \beta_4 + \cdots,
$$

(20)

where $\beta' = \frac{d\beta}{d\omega}$. From [2] we have

$$
\langle \tau \rangle_t = \langle [\Phi'(\omega) + \beta'(\omega)L] \rangle_\omega,
$$

(21)

$$
\langle \tau^2 \rangle_t = \Delta \tau^2_L + \langle [\Phi'(\omega) + \beta'(\omega)L]^2 \rangle_\omega,
$$

(22)

$$
\langle \Delta \tau^2 \rangle_t = \langle \tau^2 \rangle_t - \langle \tau \rangle_t^2,
$$

(23)

where the symbols $\langle \ldots \rangle_t$ and $\langle \ldots \rangle_\omega$ correspond to time and spectral average, respectively. Now substituting equations (21) and (22) in equation (23) we get

$$
\langle \Delta \tau^2 \rangle_t = \left[ \langle (\beta'(\omega))_\omega^2 - \langle \beta'(\omega) \rangle_\omega^2 \rangle_\omega L^2 + 2\langle (\Phi'(\omega)\beta'(\omega))_\omega \rangle - \langle \beta'(\omega) \rangle_\omega \langle \Phi'(\omega) \rangle_\omega \rangle L \right]
$$

$$
+ \langle (\Phi'(\omega))^2_\omega \rangle - \langle \Phi'(\omega) \rangle_\omega^2 + \Delta \tau^2_L
$$

(24)
and by substituting equations (15) and (20) in equation (24) we have

\[
\langle \Delta \tau^2 \rangle = \langle \Delta \Omega^2 \rangle \left[ \left( \frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2 \right) L + C_2 \right]^2 + \Delta \tau_{TL}^2.
\] (25)

where \( \langle \Delta \Omega^2 \rangle \) is the second-order central moment of the pulse in the frequency domain and \( \Delta \tau_{TL}^2 \) is the second-order central moment for the TL pulse [2]. As for a Gaussian pulse envelope the FWHM quantities \( \Delta \tau, \Delta \Omega, \Delta \tau_{TL} \) are analytically related to the corresponding second-order moments by the relation [3]

\[
\langle \Delta \tau^2 \rangle = 8 \ln(2) \Delta \tau^2,
\] (26)

\[
\langle \Delta \Omega^2 \rangle = 8 \ln(2) \Delta \Omega^2,
\] (27)

\[
\Delta \tau_{TL}^2 = 8 \ln(2) \Delta \tau_{TL}^2.
\] (28)

The intensity spectrum of SPM in the usual form can be written as [3]

\[
S(\Omega) = \frac{c}{4\pi} |A(\Omega)|^2.
\] (29)

Substituting equation (12) in equation (29) the FWHM of \( S(\Omega) \) assumes the value [3]

\[
\Delta \Omega = \frac{2 \sqrt{\ln 2}}{\tau_0} \sqrt{1 + 4 \gamma^2 z^2 p^2},
\] (30)

\[
\Delta \tau_{TL} = \frac{4 \ln 2}{\Delta \Omega}.
\] (31)

Now by substituting equations (26), (27), (28), (30), and (31) in equation (25), the following result is obtained:

\[
\Delta \tau^2 = 4 \ln 2 \left[ \tau_o^2 + \frac{\left( \frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2 \right)^2 L^2}{\tau_o^2} \right] + 4 \left( \frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2 \right) L \gamma z P
\]

\[
+ 4 \left( \frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2 \right)^2 L^2 \gamma^2 z^2 P^2.
\] (32)

The coordinates of equation (32) in its minimum with respect to \( P \) for a fixed \( L \) assumes the TL values

\[
P_{min} = -\frac{1}{2} \left( \frac{\tau_o^2}{\frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2} \right) L \gamma z,
\] (33)

\[
\Delta \tau_{min} = -2 \sqrt{\ln 2} \left( \frac{\frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2}{\tau_o} \right) L.
\] (34)
The value $\Delta t_{\text{min}} \cdot \Delta v_{\text{min}}$ can be obtained from equations (30) and (34):

$$\Delta t_{\text{min}} \cdot \Delta v_{\text{min}} = \frac{2\sqrt{\ln 2}}{\pi} \sqrt{1 + \left(\frac{1}{3} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2\right)^2 L^2},$$

and $L_{\text{min}}$ can be obtained from equation (25):

$$L_{\text{min}} = -\frac{C_2}{\frac{3}{4} \beta_4 \omega^2 + \beta_3 \omega + 2 \beta_2}.$$

### Results and Discussion

Referring to the ITU-T recommendations [10], assuming $\lambda = 1.55 \mu m$, $\frac{\partial \tau}{\partial n} = 20$ ps/nm.km, and $D' = \frac{d^2 \tau}{d \lambda^2} = 0.085$ ps/nm².km, we obtain the following dispersion parameters using equations (17)-(19): $\beta_2 = \frac{d^2 \beta}{d \omega^2} = -25.44$ (ps)²/km, $\beta_3 = \frac{d^3 \beta}{d \omega^3} = 0.179$ (ps)³/km, $\beta_4 = \frac{d^4 \beta}{d \omega^4} = -0.001277$ (ps)⁴/km, $n_2 = 2.6 \times 10^{-20} m^2$, $f = 76$ MHz, and $\tau_o = 4.518$ ps.

By using equation (36), the value of $L_{\text{min}}$ is obtained for different combinations of dispersion terms as presented in Table 1. It is apparent from Table 1 that if we ignore all the second- and higher-order dispersion effects at the initial stage, then the value of $L_{\text{min}}$ obtained is 23 m, which agrees with the existing results [3]. Now if the second- and higher-order dispersion effects are taken into account, the value of $L_{\text{min}}$ is obtained to be 11.5 m. If the first-order dispersion effects are compensated for, then $L_{\text{min}}$ changes from 11.5 m to $6.8161 \times 10^3$ km. Further, if the first- and second-order dispersion compensation is made together, then this length changes from 11.5 m to $6.0343 \times 10^9$ km.

Now by using equation (35) for different dispersion terms, the value of the minimum pulse width and linewidth product ($\Delta t_{\text{min}} \cdot \Delta v_{\text{min}}$) is obtained with different levels of compensations, as shown in Table 2. If we ignore the dispersion effects, then the value of minimum pulse width and linewidth product obtained is 0.44, which is same as obtained in [3]. If we consider the first-, second-, and higher order dispersion effects, then the value is calculated to be 0.4418. If the first-order dispersion compensation is made, then it is 0.4411, and if the first- and second-order dispersion compensation is performed, then the value remains unchanged, that is, 0.4411, indicating the lesser impact of second- and higher-order dispersion on the minimum pulse width and linewidth product.

### Table 1

<table>
<thead>
<tr>
<th>Dispersion compensation</th>
<th>Transmission distance $L_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_2$ only as by [3]</td>
<td>23 m</td>
</tr>
<tr>
<td>No compensation</td>
<td>11.5 m</td>
</tr>
<tr>
<td>$\beta_2$ compensated</td>
<td>$6.8161 \times 10^3$ km</td>
</tr>
<tr>
<td>$\beta_2$ and $\beta_3$ compensated</td>
<td>$6.0343 \times 10^9$ km</td>
</tr>
</tbody>
</table>
Table 2
Effect of higher-order dispersion compensation on pulse width and linewidth product

<table>
<thead>
<tr>
<th>Dispersion compensation</th>
<th>$\Delta t_{\text{min}} \cdot \Delta v_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ignore dispersion effects as in [3]</td>
<td>0.44</td>
</tr>
<tr>
<td>No compensation</td>
<td>0.4418</td>
</tr>
<tr>
<td>$\beta_2$ compensated</td>
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</tr>
<tr>
<td>$\beta_2$ and $\beta_3$ compensated</td>
<td>0.4411</td>
</tr>
</tbody>
</table>

Conclusions

In this paper we have shown the influence of higher-order effects of dispersion on the femto-second transform limited pulse generation by compensating for linear chirp of self-phase modulation spectra in the dispersion-shifted fibers. It has been shown that the minimum propagation length with first-order dispersion term is 23 m, as reported earlier. If the higher-order dispersion effects are taken into consideration, this length is reduced to 11.5 m. With the first- and first- and second-order dispersion compensation together, this length can be enhanced to $6.8161 \times 10^3$ and $6.0343 \times 10^9$ km, respectively. The minimum pulse width and linewidth product without dispersion, with dispersion including higher-order dispersion effects, and with dispersion compensation is found to be 0.44, 0.4418, and 0.4411, respectively.

References


**Biographies**

Harish Kumar obtained his Bachelor’s degree in electronics engineering from Punjab Technical University, Jalandhar, Punjab, India, and M.Tech. degree from Punjab University, Chandigarh, India, in 1998 and 2002, respectively. He has been working as a lecturer in the Electronics and Communication Engineering Department, SLIET, Longowal, Punjab, India since 2000. His present interests are fiber dispersion and nonlinearities. He has published four papers in international journals. He is a life associate member of the Institution of Engineers (India).

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Ajay K. Sharma received his B.E. degree in electronics and electrical communication engineering from Punjab University, Chandigarh, India, in 1986. His M.S. degree is in electronics and control engineering from the Birla Institute of Technology and Science, Pilani, in 1994, and his Ph.D. is in electronics, communication, and computer engineer-
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