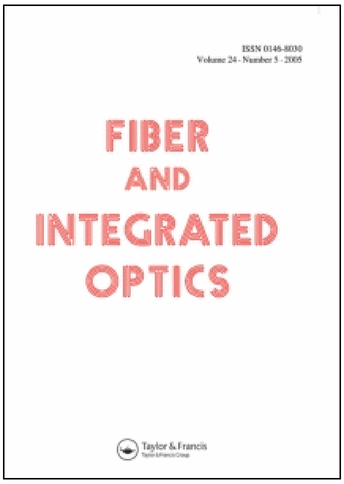


Fiber and Integrated Optics
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713771194

Approximate and Exact Small Signal Analysis for Single-Mode Fiber Near Zero Dispersion Wavelength with Higher Order Dispersion

R. S. Kaler; T. S. Kamal; Ajay K. Sharma

Online publication date: 10 November 2010

To cite this Article Kaler, R. S. , Kamal, T. S. and Sharma, Ajay K.(2002) 'Approximate and Exact Small Signal Analysis for Single-Mode Fiber Near Zero Dispersion Wavelength with Higher Order Dispersion', Fiber and Integrated Optics, 21: 5, 391 — 415

To link to this Article: DOI: 10.1080/01468030290096859

URL: http://dx.doi.org/10.1080/01468030290096859

Fiber and Integrated Optics, 21:391–415, 2002

0146-8030/02 $12.00 + .00
DOI: 10.1080/0146803029009685 9

Approximate and Exact Small Signal Analysis for Single-Mode Fiber Near Zero Dispersion Wavelength with Higher Order Dispersion

R. S. KALER
T. S. KAMAL

Department of Electronics & Communication Engineering
Sant Longowal Institute of Engineering & Technology
Longowal-148106, Distt. Sangrur, Punjab, India

AJAY K. SHARMA

Department of Electronics & Communication Engineering
Regional Engineering College
Jalandhar-144011, Punjab, India

This article presents the comparison of approximate and exact small-signal theories for analyzing the influence of the higher-order dispersion terms on dispersive optical communication systems operating near zero dispersion wavelength for linear single-mode fiber. For the approximate theory, the generalized conversion matrix has been reported and gives the transfer function of intensity and phase from the fiber input to fiber output for a laser source including the influence of any higher-order dispersion term. In addition, expressions for the small-signal frequency response and the relative intensity noise (RIN) response of an ultrafast laser diode including noises are derived. However, it is observed that the approximation assumed for the second-order dispersion term for the approximate analysis is not valid. From the approximate theory, the exact generalized conversion matrix and exact expressions for small-signal frequency response and relative intensity noise (RIN) are obtained. We show that for the exact theory, the second-order dispersion term has no effect on intensity and frequency response even at large modulating frequencies and large propagation distances contrary to the approximate theory as reported by other authors. But we show that third-order dispersion term certainly has some minute impact on the frequency and RIN response for long distance links at high modulating frequencies.

Keywords broadband communications, dispersion, laser noise, relative intensity noise, higher-order dispersion, frequency response

Received 22 February 2002; accepted 3 April 2002.

The authors would like to thank the Ministry of Human Resource Development (MHRD), Govt. of India, New Delhi, for financial support for the work under the Project *Studies on Dispersion and Fiber Nonlinearities on Broadband Optical Communication Systems and Networks*.

Address correspondence to R. S. Kaler, Assistant Professor, (ECE), Sant Longowal Institute of Engineering and Technology, Sangrur, Punjab, India. E-mail: rskaler@yahoo.com

Introduction

The invention of the erbium-doped fiber amplifier (EDFA) [1, 2] paved the way for the development of high bit rate all optical ultra-long-distance communication systems. Specifically, periodic compensation of fiber loss by EDFAs eliminates the need for electronic repeaters along the transmission line and enables the construction of all-optical communication systems in which the transmission distance is limited by the fiber chromatic dispersion rather than by the fiber loss [3] because it introduces signal distortion and noise [4, 5]. However, if conventional 1.3 μm zero dispersion optical fiber systems and networks are used for the 1.55 μm signal light, they exhibit a significant dispersion yielding, e.g., limitations with respect to transmission bandwidth [6]. There is also signal degradation due to nonlinear effects [7] like Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM), Four-Wave Mixing (FWM), Stimulated Raman Scattering (SRS) and Stimulated Brillouin Scattering (SBS). However, in a well-designed system these effects can be minimized. SBS can be avoided by broadening the carrier component of the signal. The effects of SPM and XPM can be reduced by appropriate use of dispersion-shifted fibers. Degradations caused by FWM can also be reduced by the effects of dispersion.

To increase the transmission capacity of the systems, several techniques based on fiber nonlinearities have been studied [8]. In particular, several authors have demonstrated experimentally the propagation of optical solitons without distortion through very long fiber links. Nevertheless, experiments and theory have shown that the realization of such nonlinear optical transmission systems over very long distances is very expensive [7, 8]. Moreover, it has also been shown that it is very difficult to describe the propagation of the optical signal near zero first-order dispersion wavelength especially taking into account the effects of nonlinearities [9, 10]. It is therefore very useful and attractive to investigate the ultimate performance of traditional systems in the linear regime near zero dispersion wavelength because of its simplicity and because it can still be applied to realize broadband optical communication systems.

Wang and Petermann [11] developed a new approach to investigate the influence of the dispersion on optical fiber communication systems using small-signal analysis near zero dispersion wavelength. The conversion between phase and intensity modulation or noise caused by chromatic dispersion has been analyzed for laser diodes. A conversion matrix describing the transfer function of intensity and frequency modulation at fiber input to the intensity and frequency modulation at fiber output was reported, and the results were obtained to analyze the performance of optical communication systems with first-order dispersion.

A theory describing the propagation of signal and noise through a lossless linear dispersive single-mode fiber with first- and second-order dispersion was presented by Crognale [12] near zero dispersion wavelength. Recalling the small-signal approach reported in Wang and Peterman [11], a simple and exhaustive treatment was developed to study the small-signal and noise transmission characteristics in the frequency domain of a high performance laser diode together with a linear dispersive fiber. The impact of the second-order dispersion term on the modulation and noise properties of an ultrafast laser diode was obtained to study the total frequency response and the relative intensity noise (RIN) at output of the linear single-mode fiber, but the impact of third- and higher-order dispersion terms was neglected. It was shown that the second-order dispersion term had significant impact for long distance links at high modulating frequencies.

Cartaxo et al. [13] further carried out rigorous small-signal analysis using theoretical and numerical simulations for linear dispersive optical communication systems operating near zero wavelength. The theory contradicted the Crognale theory and indicated that the approximation assumed in Crognale [12] was not valid and the second-order dispersion term had no impact on frequency and RIN response for long distance links at high modulating frequencies. However, the analysis was again carried up to the second-order dispersion term only.

In this article, we have extended the work reported above [11–13] by presenting theoretical analysis for analyzing the influence up to the third-order dispersion term on a dispersive optical communication system. We consider the theory reported in Crognale [12] as approximate analysis and theory indicated by Cartaxo and Morgado [13] as exact analysis. We extend the approximate analysis up to a third- or fourth-order dispersion term and present a more general small-signal relation allowing conversion between intensity and phase modulation or noise in a dispersive fiber including higher order dispersion terms. For this theory, a modified conversion matrix has also been reported that gives the transfer function of intensity and phase from the fiber input to the fiber output for any laser source including the influence of any high-order dispersion term. Moreover, this theory is applicable to evaluate the impact of higher-order dispersion on the small-signal frequency response and RIN of an ultrafast laser diode similarly as mentioned in Crognale [12]. Considering the approximation reported in Cartaxo and Morgado [13], we converted the approximate analysis into exact analysis and obtained the same results up to a second-order dispersion. But on carrying the analysis further, we show that the third-order dispersion term certainly has some impact on the frequency and RIN response for long distance links at high modulating frequencies. We will present in the next section the modified analysis up to fourth-order dispersion term for a phase and intensity of modulated signal propagating through a dispersive medium, yielding the desired generalized conversion matrix inclusive influence of higher-order dispersion terms. In the section on approximate small signal analysis we discuss the application of a conversion matrix to the approximate small-signal intensity and frequency modulation characteristics of a laser diode; in the section on approximate small-signal characteristics and RIN we discuss approximate small signal RIN at output of a dispersive fiber including the impact of higher-order dispersion. In the last two sections, respectively, exact expressions for frequency and RIN response are obtained for exact analysis and the various responses are plotted, and results are obtained.

Analysis for Intensity and Phase of an Optical Field at the Fiber Output

In this section, we present a transfer function using an approach similar to that reported in Wang and Petermann, and Crognale [11, 12] for a linear dispersive single-mode fiber, including the effects of the higher-order dispersion terms; i.e., first-, second-, third-, and fourth-order dispersion terms. In fact, the chromatic dispersion is the most significant limiting factor to degrade the performance of ultrafast long-distance broadband optical communication systems. Therefore, it is important to analyze not only the impact of first-order dispersion, but also its slope (second-order dispersion, 2 OD), curvature (third-order dispersion, 3 OD) and quadvature (fourth-order dispersion, 4 OD) for ultrafast long distance broadband optical communication systems. Let the electric field at the input of fiber from a single mode laser diode [14]

$$E(t) = E_{in}(t)e^{j\omega_o t} \tag{1}$$

with the slowly varying complex amplitude, $E_{in}(t)$, and the mean optical frequency, ω_o, which is given by [11]:

$$E_{in}(t) = \sqrt{S_{in}(t)}e^{j\phi_{in}(t)}, \tag{2}$$

where $S_{in}(t)$ and $\phi_{in}(t)$ are, respectively, the input photon intensity and the input phase. As in Anders and Sahlen [15], the propagation of the signal through an optical fiber can be described by propagation term $e^{-j\beta L}$ with length L of the transmission fiber and the propagation constant β by the relation

$$E_{out}(\omega) = E_{in}(\omega)e^{-j\beta L}. \tag{3}$$

The losses are neglected here as the signal distortion or noise is induced by chromatic dispersion rather than fiber loss as reported in [11–13]. The propagation constant β in terms of Taylor series can be expanded around $\omega = \omega_o$ as [15–17]

$$\begin{aligned}\beta = \beta_o + (\omega-\omega_o)\tau + \frac{1}{2}(\omega-\omega_o)^2\frac{d\tau}{d\omega} + \frac{1}{6}(\omega-\omega_o)^3\frac{d^2\tau}{d\omega^2}\\ + \frac{1}{24}(\omega-\omega_o)^4\frac{d^3\tau}{d\omega^3} + \frac{1}{120}(\omega-\omega_o)^5\frac{d^4\tau}{d\omega^4}\ldots,\end{aligned} \tag{4}$$

where $\frac{d\beta}{d\omega} = \tau$ is the group delay for unit length,

$$\frac{d\tau}{d\omega} = -\frac{\lambda^2}{2\pi c}\frac{\partial\tau}{\partial\lambda} \tag{5}$$

is first-order dispersion,

$$\frac{d^2\tau}{d\omega^2} = \frac{\lambda^2}{(2\pi c)^2}\left[\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 2\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{6}$$

is second-order dispersion,

$$\frac{d^3\tau}{d\omega^3} = -\frac{\lambda^3}{(2\pi c)^3}\left[\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 6\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 6\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{7}$$

is third-order dispersion, and

$$\frac{d^4\tau}{d\omega^4} = \frac{\lambda^4}{(2\pi c)^4}\left[\lambda^4\frac{\partial^4\tau}{\partial\lambda^4} + 12\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 36\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 24\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{8}$$

is fourth-order dispersion.

Recalling Equation (3), the following expression is obtained for the propagation constant term:

$$e^{-j\beta L} = e^{-j\left(\beta_o+(\omega-\omega_o)\tau+\frac{1}{2}(\omega-\omega_o)^2\frac{d\tau}{d\omega}+\frac{1}{6}(\omega-\omega_o)^3\frac{d^2\tau}{d\omega^2}+\frac{1}{24}(\omega-\omega_o)^4\frac{d^3\tau}{d\omega^3}+\frac{1}{120}(\omega-\omega_o)^5\frac{d^4\tau}{d\omega^4}\cdots\right)L}, \tag{9}$$

where $\phi_o = \beta_o L$ at $\omega = \omega_o$. As reported in [15–17], we neglect the absolute phase ($\phi_o = \beta_o L$), and group delay ($d\beta/d\omega = \tau$ and corresponds to the F_1 term), because both terms produce only phase delay of the carrier signal and have no influence on the distortion of the signal. We define the following dispersion parameters:

$$F_2 = -\frac{L}{2}\frac{d\tau}{d\omega} = \frac{L}{2}\frac{\lambda^2}{2\pi c}\frac{\partial\tau}{\partial\lambda} \tag{10}$$

for first-order dispersion,

$$F_3 = \frac{L}{6}\frac{d^2\tau}{d\omega^2} = \frac{L}{6}\frac{\lambda^2}{(2\pi c)^2}\left[\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 2\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{11}$$

for second-order dispersion,

$$F_4 = -\frac{L}{24}\frac{d^3\tau}{d\omega^3} = \frac{L}{24}\frac{\lambda^3}{(2\pi c)^3}\left[\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 6\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 6\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{12}$$

for third-order dispersion, and

$$F_5 = \frac{L}{120}\frac{d^4\tau}{d\omega^4} = \frac{L}{120}\frac{\lambda^4}{(2\pi c)^4}\left[\lambda^4\frac{\partial^4\tau}{\partial\lambda^4} + 12\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 36\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 24\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{13}$$

for fourth-order dispersion.

Therefore, in frequency domain, substituting Equations (10)–(13) in Equation (9) and then in Equation (3),

$$E_{out}(j\omega) = E_{in}(j\omega)e^{jF_2(\omega-\omega_o)^2 - jF_3(\omega-\omega_o)^3 + jF_4(\omega-\omega_o)^4 - jF_5(\omega-\omega_o)^5 \ldots}. \tag{14}$$

In time domain, since $\left(j\omega = \frac{\partial}{\partial t}\right)$, $\left((j\omega)^2 = -\omega^2 = \frac{\partial^2}{\partial t^2}\right)$, and $\left((j\omega)^3 = -j\omega^3 = \frac{\partial^3}{\partial t^3}\right)$, etc.,

$$E_{out}(t) = e^{j\omega_o t}e^{\left(-jF_2\frac{\partial^2}{\partial t^2} + F_3\frac{\partial^3}{\partial t^3} - jF_4\frac{\partial^4}{\partial t^4} + F_5\frac{\partial^5}{\partial t^5}\ldots\right)}\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}. \tag{15}$$

Let

$$E_{out}(t) = E_{in}(t) + \Delta E(t), \tag{16}$$

where

$$|\Delta E(t)| \ll |E_{in}(t)|. \tag{17}$$

From Equations (15) and (16)

$$\Delta E(t) = \left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2} + F_3\frac{\partial^3}{\partial t^3} - jF_4\frac{\partial^4}{\partial t^4} + F_5\frac{\partial^5}{\partial t^5}\ldots\right)} - 1\right)\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}. \tag{18}$$

From Equation (16) and with the help of the approximation in Equation (17), we have

$$S_{out}(t) = |E_{in}(t) + \Delta E(t)|^2 \approx |E_{in}(t)|^2 + 2\Re[E_{in}^*(t).\Delta E(t)]. \tag{19}$$

Substituting Equations (2) and (18) in Equation (19),

$$S_{out} = S_{in}(t) + 2\Re\left[\sqrt{S_{in}(t)}e^{-j\phi_{in}(t)}\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)}-1\right)\right. \\ \left.\cdot\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\right], \tag{20}$$

$$\therefore \phi_{out}(t) = \phi_{in}(t) + \Im\left[\frac{\Delta E(t)}{E_{in}(t)}\right], \tag{21}$$

$$\phi_{out}(t) = \phi_{in}(t) + \Im\left[\frac{\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)}-1\right)\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}}{\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}}\right]. \tag{22}$$

Equations (20) and (22) derived above are the general equations describing the intensity and phase of an optical field after propagating through a dispersive optical fiber. These equations are valid for arbitrary input intensity and phase as long as dispersion induced field amplitude, $\Delta E(t)$, is assumed to be small compared to the input field, $E_{in}(t)$. Further, Equations (20) and (22) can be simplified using small-signal analysis.

Approximate Small-Signal Analysis

The small-signal analysis implies that fluctuations $\Delta S_{in}(t)$ (including fluctuations with frequency modulation or noise $\dot{\phi} = d\phi/dt$) are smaller than the average intensity $\langle S\rangle$ in the total signal $S_{in}(t)$.

$$S_{in}(t) = \langle S\rangle + \Delta S_{in}(t), \tag{23}$$

with $\langle S\rangle \gg \Delta S_{in}(t)$.

As reported in Wang and Petermann and Crognale [11, 12] in the small-signal approach, the field amplitude $\sqrt{S_{in}(t)}$ can be linearized as

$$\sqrt{S_{in}t} \approx \sqrt{\langle S\rangle}\left(1 + \frac{\Delta S_{in}(t)}{2\langle S\rangle}\right). \tag{24}$$

After neglecting the product of small-signal [11, 12, 20], we can introduce following approximation:

$$\frac{\partial^n}{\partial t^n}(\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}) \approx \sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\frac{\partial^n}{\partial t^n}\sqrt{\langle S\rangle}\left(j\phi_{in}(t) + \frac{\Delta S_{in}(t)}{2\langle S\rangle}\right). \tag{25}$$

Inserting Equation (25) into Equations (20) and (22), we obtain

$$S_{out}(t) = S_{in}(t) + 2\Re\left[\langle S\rangle\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)}\right)\left(\frac{\Delta S_{in}(t)}{2\langle S\rangle} + j\phi_{in}(t)\right)\right], \tag{26}$$

with $S_{in}(t) \approx \langle S \rangle$ as Equation (23) with $\langle S \rangle \gg \Delta S_{in}(t)$ and similarly as

$$\phi_{out}(t) = \Im\left[\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)}\right)\left(\frac{\Delta S_{in}(t)}{2\langle S \rangle} + j\phi_{in}(t)\right)\right]. \tag{27}$$

The exponential operator can be written as [12]

$$\begin{aligned} e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)} &\approx e^{-j\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)} e^{\left(F_3\frac{\partial^3}{\partial t^3}+F_5\frac{\partial^5}{\partial t^5}\right)} \\ &\approx e^{-j\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)}\left(1 + F_3\frac{\partial^3}{\partial t^3} + F_5\frac{\partial^5}{\partial t^5}\right). \end{aligned} \tag{28}$$

We use approximation assuming F_3 and F_5 to be very small and expressing $e^x = 1+x$. (The expansion has been carried out only up to the first term, the higher-order terms being ignored. This is because the intensity spectrum caused by noise is considered to be narrow. The products of F_3 and F_5 and their squares are small and further $F_3^2 < F_5$.) Also since $e^{-jx} = \cos(x) - j\sin(x)$, we get

$$\begin{aligned} &e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}\cdots\right)} \\ &\approx \left[\cos\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right) - j\sin\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right)\right]\left(1 + F_3\frac{\partial^3}{\partial t^3} + F_5\frac{\partial^5}{\partial t^5}\right). \end{aligned} \tag{29}$$

In this way the relations for intensity and phase derived at the fiber output are:

$$\begin{aligned} \Delta S_{out}(t) = {} & \cos\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right)\left[\Delta S_{in}(t) + F_3\Delta \dddot{S}_{in}(t) + F_5\Delta \overset{\cdot\cdot\cdot\cdot\cdot}{S}_{in}(t)\right] \\ & + 2\langle S \rangle \sin\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right)\left[\phi_{in}(t) + F_3\dddot{\phi}_{in}(t) + F_5\overset{\cdot\cdot\cdot\cdot\cdot}{\phi}_{in}(t)\right], \end{aligned} \tag{30}$$

$$\begin{aligned} \phi_{out} = {} & \cos\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right)\left[\phi_{in}(t) + F_3\dddot{\phi}_{in}(t) + F_5\overset{\cdot\cdot\cdot\cdot\cdot}{\phi}_{in}(t)\right] \\ & - 2\sin\left(F_2\frac{\partial^2}{\partial t^2} + F_4\frac{\partial^4}{\partial t^4}\right)\left[\frac{\Delta S_{in}(t)}{2\langle S \rangle} + F_3\frac{\Delta \dddot{S}_{in}(t)}{2\langle S \rangle} + F_5\frac{\Delta \overset{\cdot\cdot\cdot\cdot}{S}_{in}(t)}{2\langle S \rangle}\right], \end{aligned} \tag{31}$$

with the output modulation or noise given by

$$\Delta S_{out}(t) = S_{out}(t) - \langle S \rangle \tag{32}$$

and dots denote the time derivatives. Recalling the relation between frequency and phases [11, 12], we take practical systems and consider frequency modulation or noise $\dot{\phi} = d\phi/dt$ rather than phase modulation. The results may be expressed with a modified conversion matrix that describes the relation between intensity and frequency including the second-, third-, and fourth-order dispersion terms as shown in the expressions in Equations (30) and (31). These expressions are the extension of the expression reported in [11, 12].

$$\begin{pmatrix} \Delta S_{out} \\ \dot{\phi}_{out} \end{pmatrix} = \begin{pmatrix} \cos(F_2\omega^2 + F_4\omega^4)(1 - jF_3\omega^3 - jF_5\omega^5) \\ 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4)\left(\frac{j}{\omega} + F_3\omega^2 + F_5\omega^4\right) \\ \frac{\sin(F_2\omega^2 + F_4\omega^4)}{2\langle S\rangle}(j\omega + F_3\omega^4 + F_5\omega^6) \\ (1 - j\omega^3 F_3 - j\omega^5 F_5)\cos(F_2\omega^2 + F_4\omega^4) \end{pmatrix} \begin{pmatrix} \Delta S_{in}(\omega) \\ \dot{\phi}_{in}(\omega) \end{pmatrix}. \quad (33)$$

The modified conversion matrix in Equation (33) may be generalized for other higher-order dispersion terms. The generalized form of the conversion matrix may be written as in Equation (34) and is valid to derive conversion matrixes reported in [11, 12] and all previous results. Also the new results can be derived by including or excluding the respective higher-order dispersion terms:

$$\begin{pmatrix} \Delta S_{out}(\omega) \\ \dot{\phi}_{out}(\omega) \end{pmatrix} = \begin{pmatrix} \cos(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n})(1 - jF_3\omega^3 - \ldots jF_{2n+1}\omega^{2n+1}) \\ 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n}) \\ \left(\frac{j}{\omega} + F_3\omega^2 + \ldots F_{2n+1}\omega^{2n}\right) \\ \frac{\sin(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n})}{2\langle S\rangle}(j\omega + F_3\omega^4 + \ldots F_{2n+1}\omega^{2n+2}) \\ (1 - j\omega^3 F_3 - \ldots j\omega^{2n+1} F_{2n+1}) \\ \cos(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n}) \end{pmatrix} \cdot \begin{pmatrix} \Delta S_{in}(\omega) \\ \dot{\phi}_{in}(\omega) \end{pmatrix}. \quad (34)$$

If we assume that only intensity modulation is present and there is no phase modulation, we can obtain the transfer function, $\cos(F_2\omega^2+F_4\omega^4)(1-jF_3\omega^3-jF_5\omega^5)$, in comparison with $\cos(F_2\omega^2)$ [11] obtained exclusive second-, third-, and fourth-order dispersion terms.

On the other hand, if we assume that only phase modulation is present, the intensity modulation, $S_{out}(\omega)$, at output of a dispersive fiber due to FM-AM conversion can be expressed as

$$\Delta S_{out}(\omega) = 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4)\left(\frac{j}{\omega} + F_3\omega^2 + F_5\omega^4\right). \quad (35)$$

Using the preceding analysis, it is easy to calculate the transfer function in the presence of either only intensity modulation or phase modulation through a linear dispersive fiber including the influence of higher-order dispersion terms. In addition, Equation (33) doesn't need the calculation of the Bessel function.

Approximate Small-Signal Intensity and Frequency Modulation Characteristics and RIN at Output of a Dispersive Single-Mode Fiber

If we consider a modulation of the injection current around the mean value $\langle I \rangle$, the small-signal response of the laser source may be obtained with small-signal modulation current ΔI, so that $|\Delta I| \ll \langle I \rangle$, yielding in frequency domain [12]

$$\Delta S_{in}(\omega) = \left(\frac{\tau_{ph}}{e}\right) H(\omega) \Delta I(\omega), \tag{36}$$

where τ_{ph} is the photon lifetime, e is the elementary charge. The small-signal modulation transfer function is given by [12].

$$H(\omega) = \frac{\omega_r^2}{(j\omega)^2 + j\omega\Gamma + \omega_r^2}. \tag{37}$$

Here, Γ is the damping rate. The relation between frequency modulation and intensity modulation (chirp) may be described by [14, 21]

$$\dot{\phi}_{in}(\omega) = \frac{\alpha}{2}(j\omega + \omega_g)\frac{\Delta S_{in}(\omega)}{\langle S \rangle}, \tag{38}$$

where α is the linewidth enhancement factor, and ω_g is the device-specific characteristic frequency. If the chirp characteristics are caused by mainly nonlinear gain, we have $\omega_g \approx \Gamma$. The intensity modulation at the fiber output can be obtained by inserting Equations (36), (37), and (38) into Equation (33):

$$\begin{aligned} \Delta S_{out} &= \left(\frac{\tau_{ph}}{e}\right)(1 - j\omega^3 F_3 - j\omega^5 F_5) \\ &\quad \cdot \left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] H(\omega)\Delta I(\omega), \end{aligned} \tag{39}$$

$$\begin{aligned} \frac{\Delta S_{out}}{\Delta I(\omega)} &= \left(\frac{\tau_{ph}}{e}\right)(1 - j\omega^3 F_3 - j\omega^5 F_5) \\ &\quad \cdot \left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] H(\omega). \end{aligned} \tag{40}$$

Equation (40) gives intensity modulation at the output of a dispersive fiber, including the effects of first-, second-, third-, and fourth-order dispersion terms. Recalling the results reported in Wang and Petermann and Crognale [11, 12], and from Equation (40) we can derive all the previous results of small-signal response:

$$\frac{\Delta S_{out}}{\Delta I(\omega)} = \left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] \left|\frac{\Delta S_{out}}{\Delta I(\omega)}\right|_{F_3=0,\, F_5=0}. \tag{41}$$

Equation (41) shows that at high modulation frequencies the second- and fourth-order dispersion terms introduce an enhancement factor,

$$\left|\frac{\Delta S_{out}}{\Delta I(\omega)}\right| = (1 + \omega^6 F_3^2 + \omega^{10} F_5^2 + 2\omega^8 F_3 F_5) \left|\frac{\Delta S_{out}}{\Delta I(\omega)}\right|_{F_2=0,\, F_4=0}. \tag{42}$$

In addition, this small-signal analysis may be applied to evaluate the chromatic dispersion on the small-signal frequency response and RIN of an ultrafast laser diode, as reported in [11, 12] in order to see the influence of higher-order dispersion terms together and independently. Similarly, we can derive and analyze the impact of other independent and combined higher-order dispersion terms for semiconductor laser noise with spontaneous emission rate and average photon density. Assuming that the laser noise is induced predominantly by the spontaneous emission noise, the Langevin noise sources for intensity and phase are given by [11, 14],

$$\langle |F_s(\omega)|^2 \rangle = 2R\langle S \rangle, \tag{43}$$

$$\langle |F_\phi(\omega)|^2 \rangle = \frac{R}{2\langle S \rangle}, \tag{44}$$

where R and $\langle S \rangle$ are spontaneous emission rate and average photon density, respectively, of the semiconductor laser diode with

$$\langle \Delta F_s(\omega) \Delta F_\phi^*(\omega) \rangle = 0. \tag{45}$$

As reported in Wang and Petermann [11], we have

$$\Delta S(\omega) = \left(\frac{\frac{1}{\tau_e'} + j\omega}{\omega_R^2} \right) H(\omega) \Delta F_s(\omega), \tag{46}$$

$$\dot{\phi}(\omega) = \frac{-\alpha \omega_R^2}{2\left(j\omega + \frac{1}{\tau_e'} \right)} \frac{\Delta S(\omega)}{\langle S \rangle} + \Delta F_\phi(\omega), \tag{47}$$

with Wang and Petermann [11] describing the features of a laser diode.

$$\frac{1}{\tau_e'} = \frac{1}{\tau_e} + \omega_R^2 \tau_{ph}. \tag{48}$$

It is useful to introduce a "relative intensity noise" (RIN) relating the intensity fluctuations, $\Delta S(\omega)$, referred to a noise bandwidth Δf, to mean intensity $\langle S \rangle$. By definition [12],

$$\frac{\text{RIN}}{\Delta f} = \frac{2\langle |\Delta S(\omega)|^2 \rangle}{\langle S \rangle^2}. \tag{49}$$

From Equation (46), the intrinsic RIN of a semiconductor laser may be written as

$$\frac{\text{RIN}}{\Delta f} = \frac{4R}{\langle S \rangle} \left(\frac{\frac{1}{\tau_e'^2} + \omega^2}{\omega_R^4} \right) |H(\omega)|^2. \tag{50}$$

It is well known that RIN is significantly enhanced after passing through dispersive fiber because of different delays of the spectra components within the spectral width of the laser and due to higher-order dispersion terms.

The FM-AM noise conversion with the intensity and phase noise of the laser source has been investigated by several researchers [18–21]. Recalling the results obtained in Wang and Petermann and Crognale [11, 12], and the modified conversion matrix, Equation (34), one finally obtains the RIN at the fiber output due to higher-order dispersion terms by inserting Equations (46) and (47) in (33) and recalling Equation (50):

$$\frac{\mathrm{RIN}}{\Delta f} = \frac{4R}{\langle S\rangle}(1 + F_3^2\omega^6 + F_5^2\omega^{10} + 2\omega^8 F_3 F_5) \cdot \left\{ \frac{1+\omega^2\tau_e'^2}{\omega_R^4\tau_e'^2}|H(\omega)|^2\cos^2(\omega^2F_2 + \omega^4F_4) + (\alpha^2|H(\omega)|^2 + 1) \cdot \frac{\sin^2(\omega^2F_2+\omega^4F_4)}{\omega^2} - \frac{\alpha\sin^2(2\omega^2F_2 + 2\omega^4F_4)}{\omega_R^2}|H(\omega)|^2 \right\}. \tag{51}$$

Results reported in Wang and Petermann and Crognale [11, 12] for intensity modulation and RIN at the fiber output may also be derived using the modified conversion matrix to analyze the impact of higher-order dispersion terms. The modified conversion matrix reported in this article permits to derive frequency and intensity modulation (or noise) at the fiber input, the corresponding frequency and intensity modulation (or noise) at the fiber output, taking into account the correlation between the phase and intensity modulation and considering the higher-order dispersion effects for any arbitrary frequency and intensity modulation at the fiber input.

Exact Small-Signal Analysis

The entire analysis in this article has been based on approximate analysis and is an extension of the theoretical analysis reported in Crognale [12]. It is based on the approximation $e^x = 1 + x$, where $x = F_3\omega^3 + F_5\omega^5$ up to the fourth-order dispersion term. However, it was pointed out in Cartaxo and Morgado [13] that this approximation is valid only if $|x| \ll 1$, which is not true. We now derive the exact conversion matrix by substituting $1 + x = e^x$ back into the modified conversion matrix in Equation (34):

$$\begin{pmatrix}\Delta S_{out}(\omega)\\ \dot{\phi}_{out}(\omega)\end{pmatrix} = \begin{pmatrix} \cos(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n})(e^{-jF_3\omega^3 - \ldots jF_{2n+1}\omega^{2n+1}}) & \frac{2j\langle S\rangle}{\omega}\sin(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n})(e^{-jF_3\omega^3 - \ldots jF_{2n+1}\omega^{2n+1}}) \\ \frac{j\omega\sin(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n})}{2\langle S\rangle}(e^{-jF_3\omega^3 - \ldots jF_{2n+1}\omega^{2n+1}}) & (e^{-jF_3\omega^3 - \ldots jF_{2n+1}\omega^{2n+1}})\cos(F_2\omega^2 + F_4\omega^4 + \ldots F_{2n}\omega^{2n}) \end{pmatrix} \cdot \begin{pmatrix}\Delta S_{in}(\omega)\\ \dot{\phi}_{in}(\omega)\end{pmatrix}. \tag{52}$$

The exact equations for small-signal frequency response and RIN are derived from this conversion matrix using the same analysis. Carrying the analysis up to the third-order dispersion term only, we derive the exact expression for frequency response from our approximated theory and substitute approximation $1 + x = e^x$ back into Equation (40):

$$\frac{\Delta S_{out}}{\Delta I(\omega)} = \left(\frac{\tau_{ph}}{e}\right)\{e^{-jF_3\omega^3}\} \cdot \left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] H(\omega). \tag{53}$$

Similarly, the exact expression for RIN from Equation (51):

$$\frac{\text{RIN}}{\Delta f} = \frac{4R}{\langle S\rangle}\{e^{jF_3\omega^3}\}\left\{\frac{1+\omega^2\tau'^2_e}{\omega_R^4\tau'^2_e}|H(\omega)|^2\cos^2(\omega^2F_2 + \omega^4F_4) + (\alpha^2|H(\omega)|^2 + 1) \cdot \frac{\sin^2(\omega^2F_2 + \omega^4F_4)}{\omega^2} - \frac{\alpha\sin^2(2\omega^2F_2 + 2\omega^4F_4)}{\omega_R^2}|H(\omega)|^2\right\}. \tag{54}$$

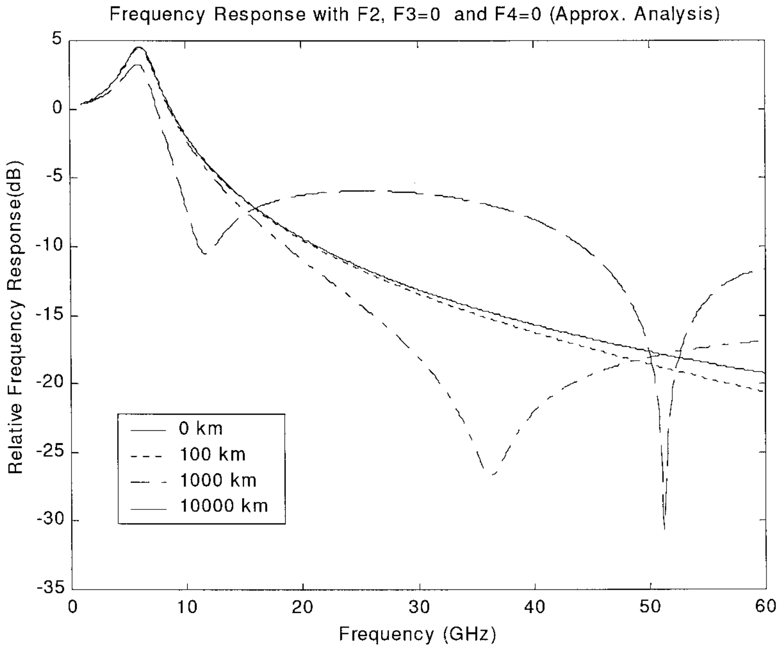
Equations (53) and (54) are the exact equations and will be used to study the impact of higher-order dispersion terms for exact analysis.

Results and Discussions

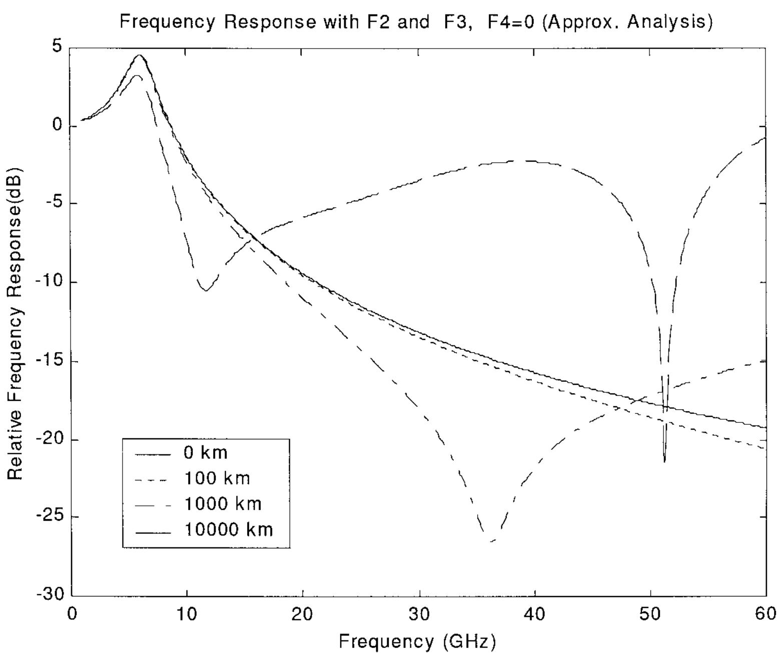
Referring to ITU:T Recommendations G.653 [22] up to the fourth-order dispersion term, we assume that for DS fiber near 1550 nm

$$\frac{d\tau}{d\lambda} = (\lambda - \lambda_o)S_o, \tag{55}$$

in which $S_o = \frac{d^2\tau}{d\lambda^2}$ is zero dispersion slope and λ_o is zero dispersion wavelength. Imposing $\lambda_o = 1550$ nm and $S_o = 0.085$ ps/nm^2/km, we have $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km. The other semiconductor laser parameters are relaxation resonance frequency $f_R = 20.25$ GHz, damping rate $\gamma = 63.29$ GHz, spontaneous emission rate $R = 2.54 \times 10^{12}$ s^{-1}, average photon density $\langle S\rangle = 4.5 \times 10^5$, photon life time $\tau_{ph} = 118 \times 10^{-12}$ s, $\tau_e = 0.17 \times 10^{-9}$ s, linewidth enhancement factor $\alpha = 5$ and maximum intrinsic modulation width $f_{\max} = 63.47$ GHz. We plot the frequency response for approximate analysis by taking Equation (40) for various different combination cases of first-, second-, and third-order dispersion terms as shown in Figure 1. Here we discuss different cases: $L = 0$, which indicates the frequency response at fiber input, $L = 100$ km, which indicates transmission distance close to practical systems, and $L = 1000$ km and $L = 10{,}000$ km, which may indicate distances of futuristic optical transmission of signals. The transmission distance $L = 10{,}000$ km has been considered as it corresponds to distance that minimizes the intensity noise (see Equation (24) in Aldolfo et al. [13]). The plot with the first-order dispersion term is shown in Figure 1a. As is clear from the figure, the frequency response with F2 varies only at large lengths from response

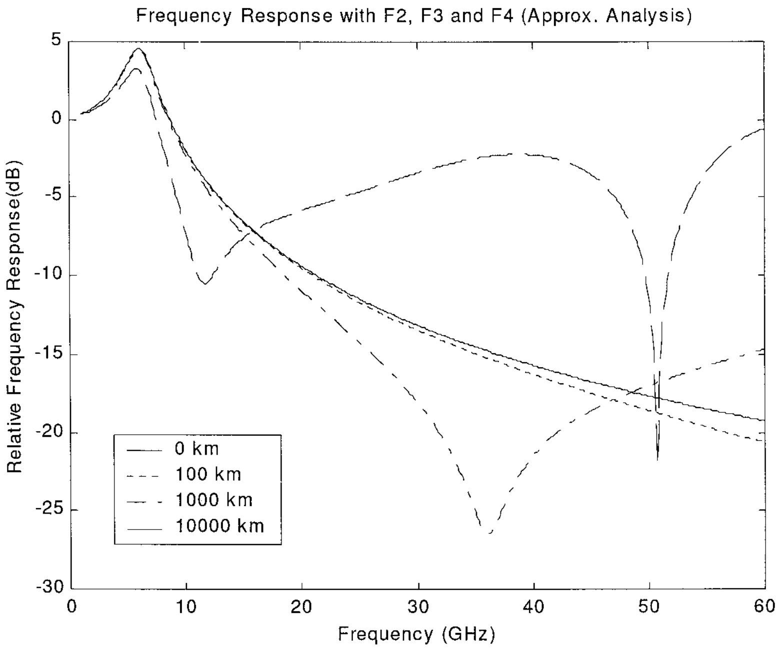


(a)

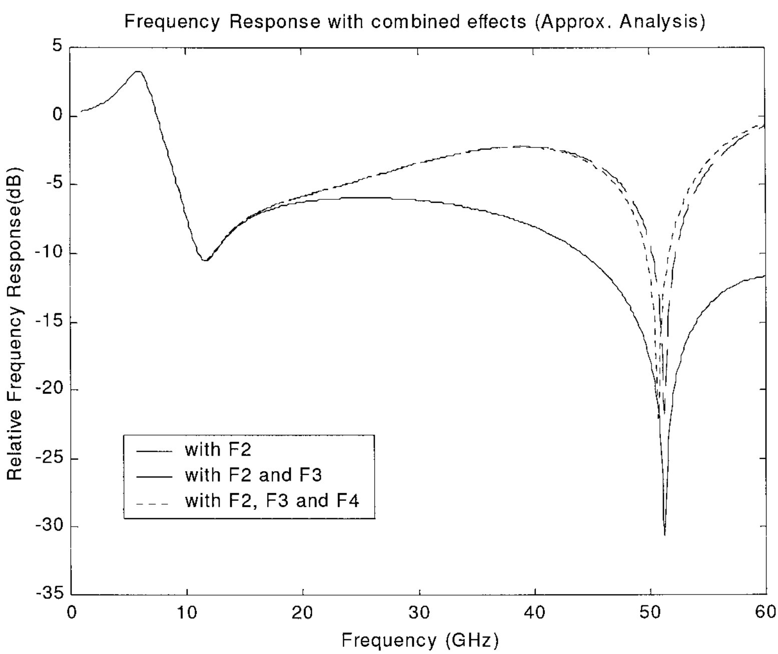


(b)

Figure 1. Approximate small-signal frequency response for different combinations of F2, F3, and F4 according to Equation (40) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (a) for F2, and (b) for F2 and F3.



(c)

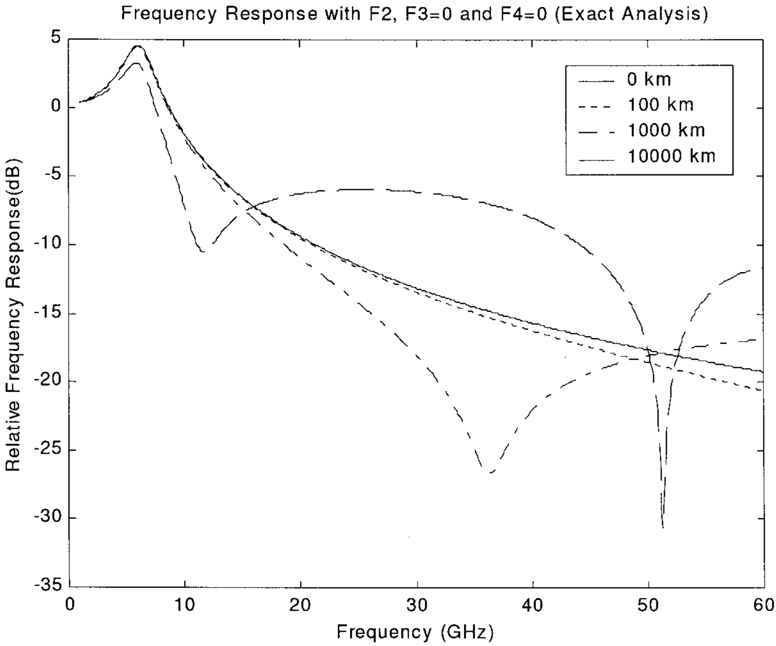


(d)

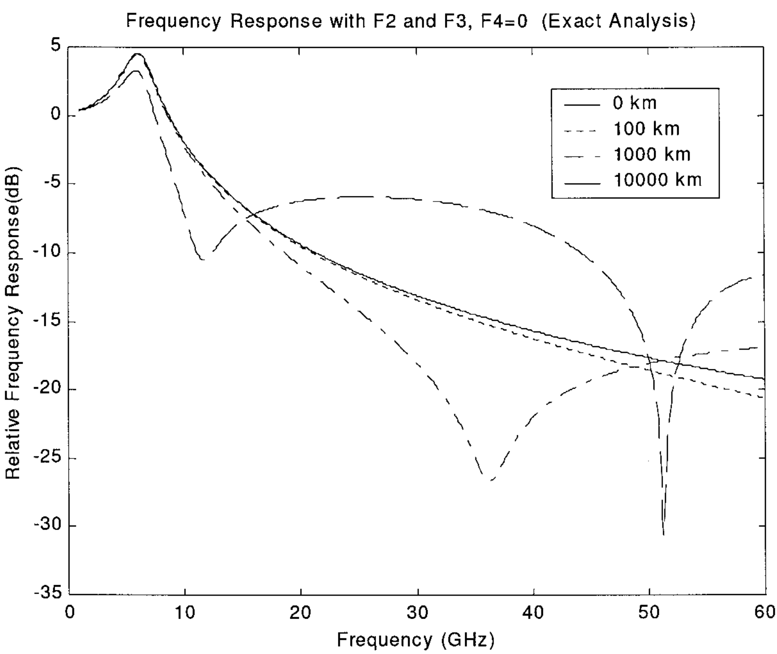
Figure 1. Approximate small-signal frequency response for different combinations of F2, F3, and F4 according to Equation (40) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (c) for F2, F3, and F4, and (d) comparison of different terms by the approximate method at $L = 10{,}000$ km.

at $L = 0$ at high modulation frequencies. The response with $L = 0$ km is almost linear over a wide range of modulating frequencies. With $L = 100$ km, there is a minute shift in response and it remains linear. With $L = 1000$ km and $L = 10{,}000$ km, it no longer remains linear. The nonlinear response for $L = 10{,}000$ km is more as compared to that with $L = 1000$ km. Figure 1b shows the response with combined effect of F2 and F3. It is observed that for lengths up to 1000 km, the deviation is almost negligible from earlier cases with F2 only (Figure 1a). However, at $L = 10{,}000$ km, the deviation is very large for this approximate analysis. With F2, F3, and F4 together, again there is minute variation for $L = 1000$ km. The variation for $L = 10{,}000$ km is still more, which indicate the additional impact of F4. Figure 1d indicates the comparison of these three cases at $L = 10{,}000$ km. It is clear from the figure that the second-order dispersion term has significant impact on frequency response at large propagation distances at long distance links for the approximate case. The results agree with the results reported in Crognale [12] for the approximate analysis. The additional impact of F4 is observed for this analysis in the figure with minute deviation as compared to the combined effect of F2 and F3. It is also seen that the drop in frequency response for F2 only and with F2 and F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at slightly low frequency. This reflects that for approximate analysis each term has its own impact on the small-signal frequency response at high modulating frequencies and large propagation distances. However these results with first- and second-order dispersion were challenged by Cartaxo, who indicated that the approximation used in the analysis was not valid. This indicates that the deviation shown for the combined effect of F2, F3, and F4 is also questionable.

In order to see the validity of the approximation, we now investigate the exact analysis obtained in the section on exact small-signal analysis. For the same set of parameters, we plot the frequency response for exact analysis by taking Equation (53) for various different combination cases of first-, second-, and third-order dispersion terms as shown in Figure 2. The same cases are taken for different lengths. The plot with the first-order dispersion term is shown in Figure 2a. We obtain the same plot as obtained in approximate analysis, which indicates that the first-order dispersion has no impact on the frequency response, as it has nothing to do with the approximation. Figure 2b shows the response with F2 and F3. The plot is the same as that obtained in the earlier case with F2 (Figure 2a), indicating that the second-order dispersion effects are irrelevant at high modulating frequencies for long distance links. We also observe also that this plot is totally different from the plot obtained in the approximate case (Figure 1b), thus challenging the approximation. These results agree with the results reported in Cartaxo and Morgado [13] for the exact analysis through numerical simulations. With F2, F3, and F4 together, again there is a drastic change from the approximate case, as shown in Figure 2c, and the plot is very close to the earlier case (Figure 2b) with F2 and F3. Still, there are minute variations that indicate the additional impact of F4 at $L = 10{,}000$ km. Figure 2d illustrates the comparison of these three cases at this distance. It is clear from Figure 2 that the second-order dispersion term has no impact on frequency response even at large propagation distances at long distance links as both the plots (with F2 only and with combined F2 and F3) coincide. But there is some impact of F4 as minute deviations are observed. The drop in frequency response for F2 only and with F2 and F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at a slightly lower frequency. Thus we conclude that the results of the approximate theory are not valid because of the approximation involved and that exact analysis is the correct one. For this analysis, there are no second-order dispersion effects, but there are

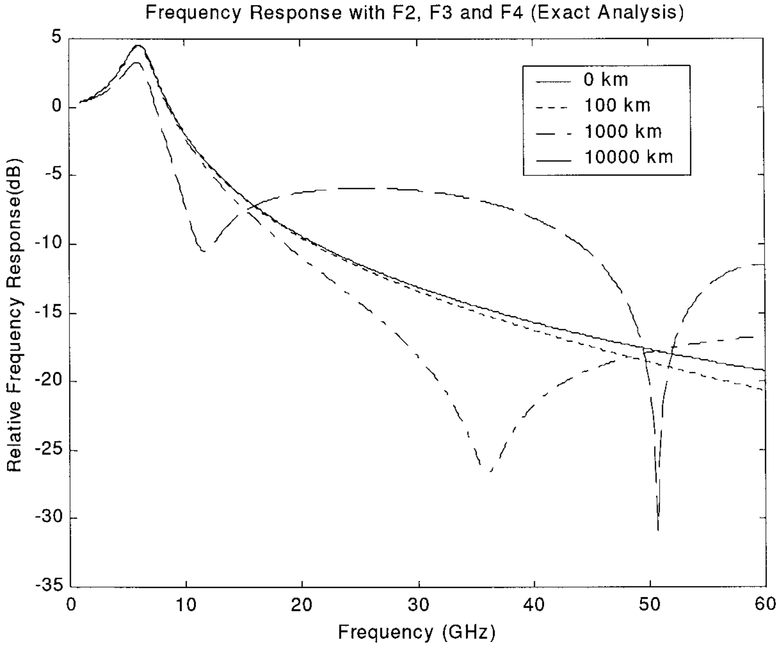


(a)

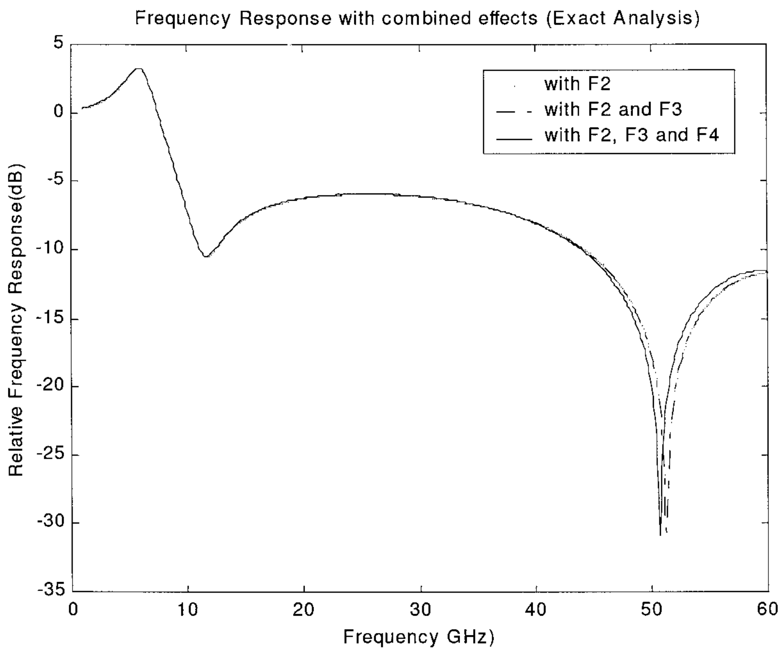


(b)

Figure 2. Exact small-signal frequency response for different combinations of F2, F3, and F4 according to Equation (53) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (a) for F2, and (b) for F2 and F3.



(c)



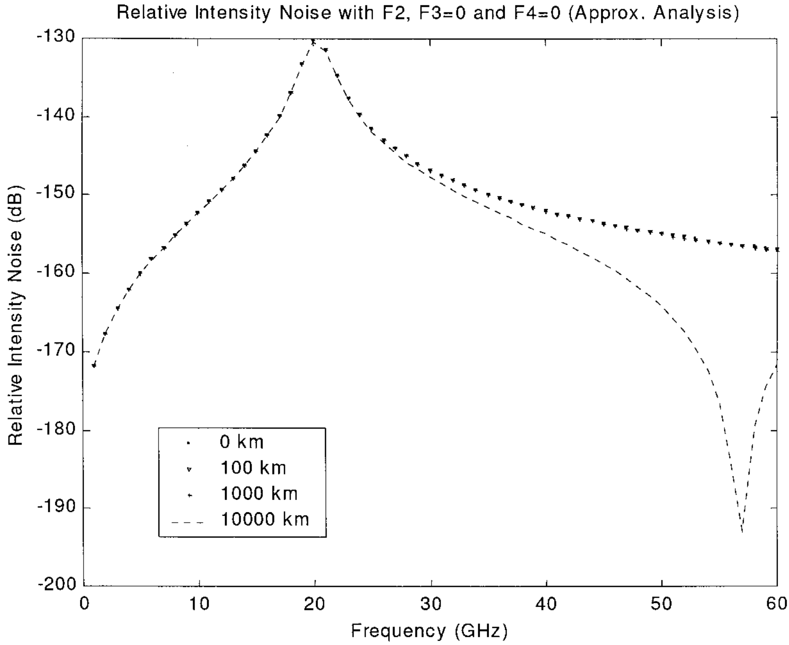
(d)

Figure 2. Exact small-signal frequency response for different combinations of F2, F3, and F4 according to Equation (53) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (c) for F2, F3, and F4, and (d) comparison of different terms by exact method at $L = 10{,}000$ km.

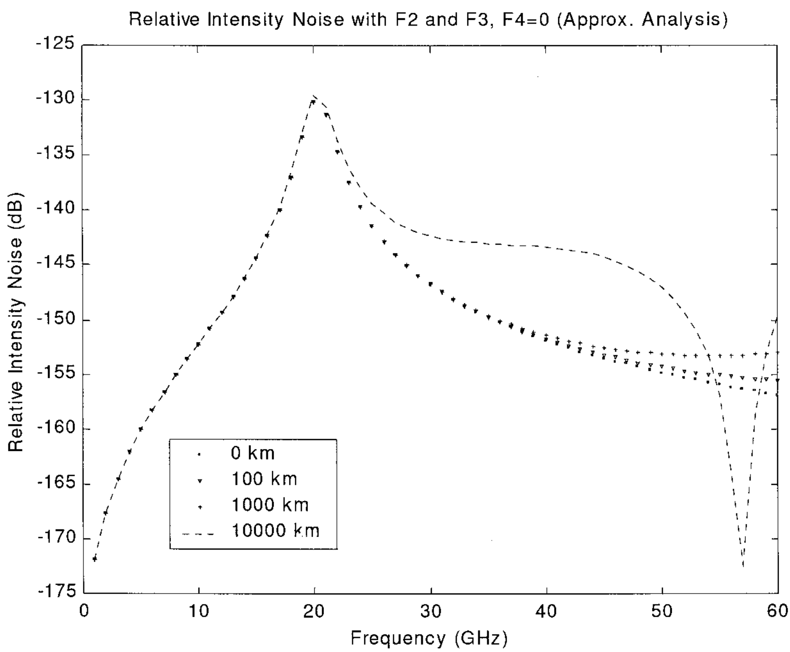
certainly some minute third-order dispersion effects at high modulating frequencies and large propagation distances.

For approximate analysis and for the same set of parameters, we now plot the Relative Intensity Noise response $\frac{\text{RIN}}{\Delta f}$ with modulation frequencies according to Equation (51) for various combination cases for first-, second-, and third-order dispersion terms as shown in Figure 3. We again discuss same cases for different lengths. The plot for RIN with first-order dispersion term is shown in Figure 3a. It is seen that the response is same up to 1,000 km but it varies for $L = 10{,}000$ km of single-mode fiber at high modulating frequencies and large propagating distances. It is also noticed that RIN response at this distance drops at approximately 57 GHz frequency and further increases with more increase in frequency. With first- and second-order dispersion terms together, the deviation is noticed from earlier case as shown in Figure 3b. It is observed that there are deviations for all propagating distances. For small distances the variations are less and vice versa. With F2, F3, and F4 together, the impact of F4 is clearly observed. The value of RIN changes for all propagation distances. Figure 3d indicates the combined impact of these three cases at $L = 10{,}000$ km. It is clear from the figure that the second-order dispersion term has a significant impact on the frequency response at large propagation distances at long distance links. The results again agree with the results reported in Crognale [12] for the approximate analysis. The additional impact of F4 is also seen in the figure for this analysis but with minute deviations. It is again seen that the drop in frequency response for F2 only and with F2 and F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at a slightly higher frequency. This indicates that for approximate analysis, each term has its own impact on the small-signal frequency response at high modulation frequencies and large propagation distances. However, the validity of these results was challenged by Cartaxo up to a second-order dispersion term and the results with combined effects of F2, F3, and F4 are regarded as questionable because of the approximation involved.

Again in order to see the validity of the approximation, we investigate the exact analysis obtained for the Relative Intensity Noise according to Equation (54). For the same set of parameters, we plot the RIN response with modulation frequencies for various combination cases of first-, second-, and third-order dispersion terms as shown in Figure 4. We again discuss the same cases of different lengths. The plot for RIN with the first-order dispersion term is shown in Figure 4a. The same plot as obtained in approximate analysis is obtained and it indicates that the first-order dispersion has no impact on RIN response as it has nothing to do with the approximation. With first- and second-order dispersion terms together, no deviation is noticed from the earlier case (F2 only), as shown in Figure 4b, but the results deviate from the approximated theory (Figure 3b). Thus the approximation involved for approximate analysis is challenged again, and we conclude that the second-order dispersion effects are irrelevant at high modulating frequencies for long distance links. These results agree with the results reported in Cartaxo and Morgado [13] for the exact analysis through numerical simulations. With F2, F3, and F4 together as shown in Figure 4c, there is drastic change from the results obtained for the approximate case (Figure 3c) and there are only some variations from the earlier case (Figure 4b) that indicate the additional impact of F4. Figure 4d shows the comparison of these three cases at $L = 10{,}000$ km. It is clear from the figure that the second-order dispersion term has no impact on frequency response even at large propagation distances at long distance links as both the plots (F2 and combined F2 and F3) coincide. But there is some impact of F4 as minute deviations are observed in comparison with the combined effect of F2 and F3. It is also seen that the drop in frequency response for F2 only and with F2

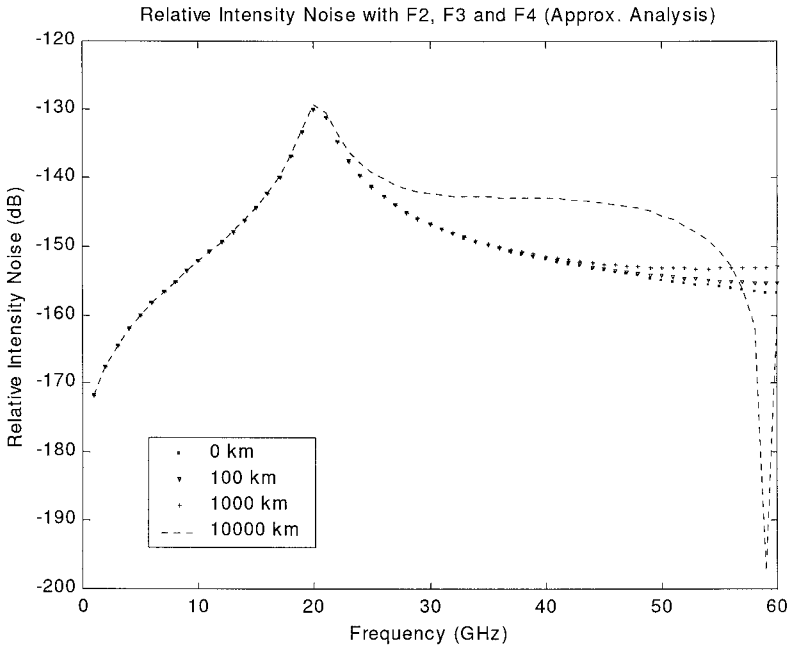


(a)

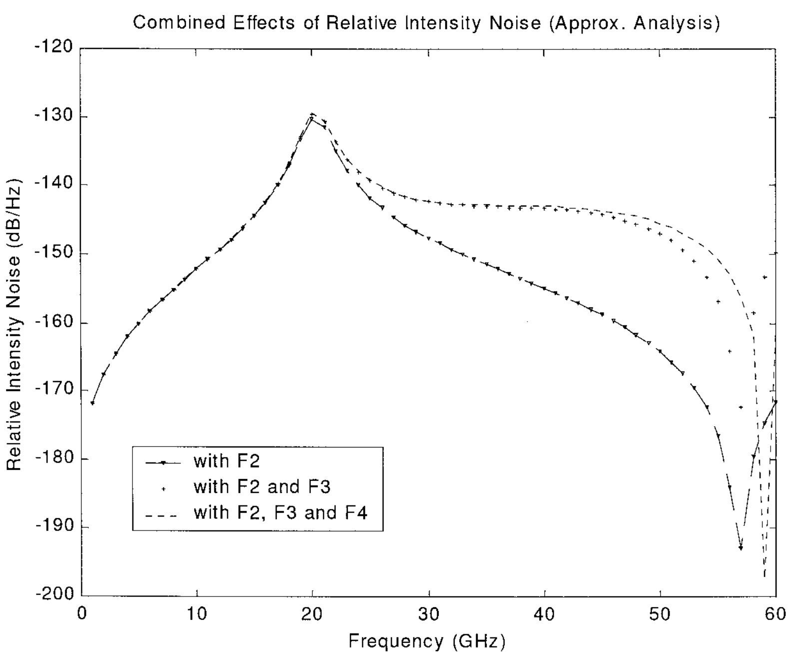


(b)

Figure 3. Approximate small-signal relative intensity noise (RIN) response for different combinations of F2, F3, and F4 according to Equation (51) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (a) for F2, and (b) for F2 and F3.

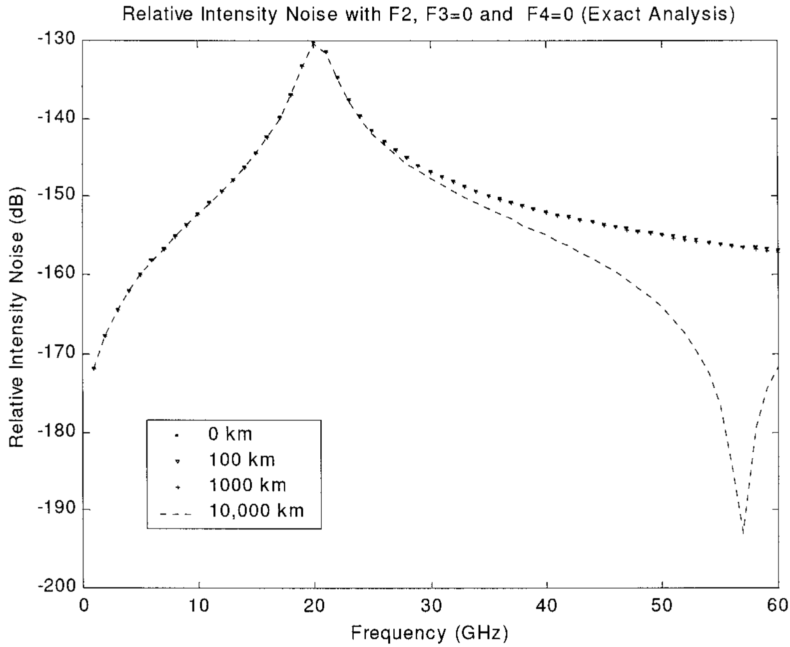


(c)

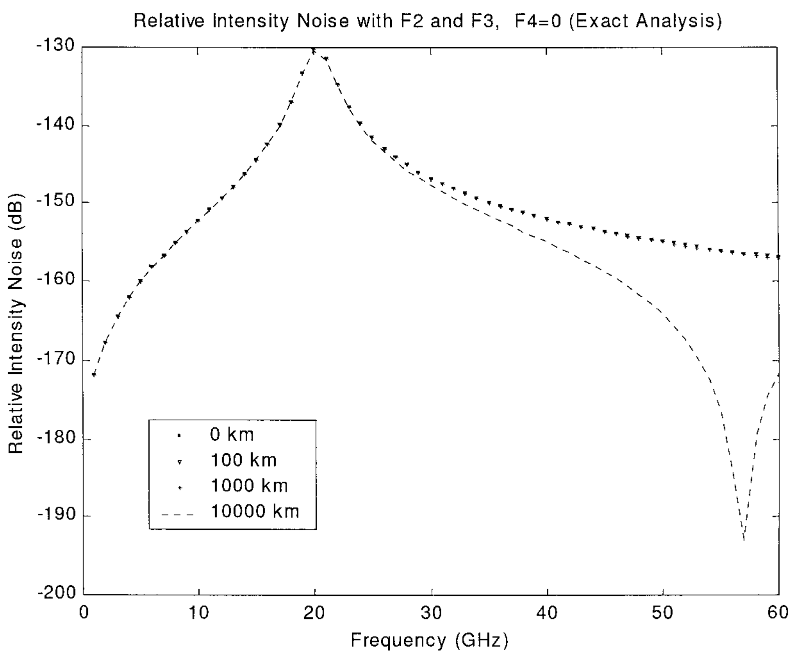


(d)

Figure 3. Approximate small-signal relative intensity noise (RIN) response for different combinations of F2, F3, and F4 according to Equation (51) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (c) for F2, F3, and F4, and (d) comparison of different terms by the approximate method at $L = 10{,}000$ km.

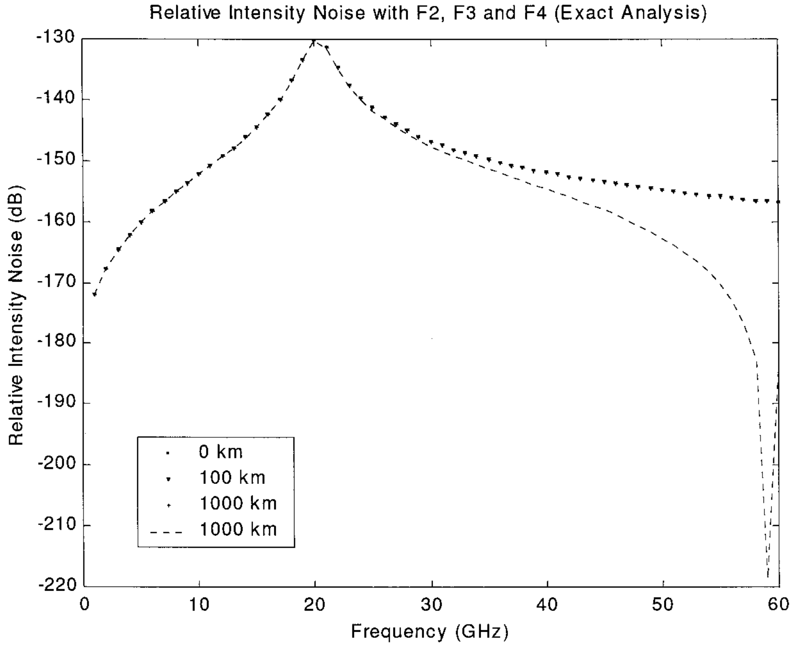


(a)

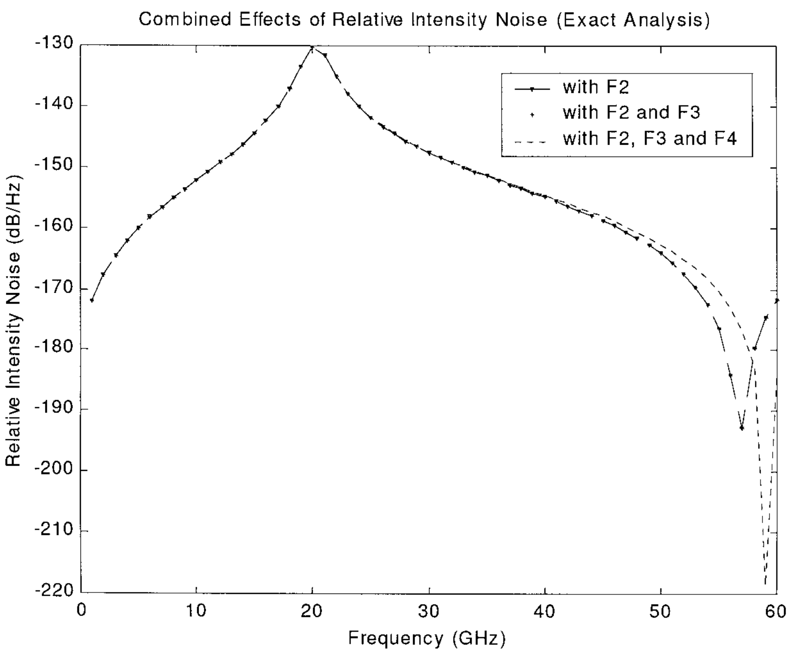


(b)

Figure 4. Exact small-signal relative intensity noise (RIN) response for different combinations of F2, F3, and F4 according to Equation (54) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (a) for F2, and (b) for F2 and F3.



(c)



(d)

Figure 4. Exact small-signal relative intensity noise (RIN) response for different combinations of F2, F3, and F4 according to Equation (54) for $L = 0$ km, $L = 100$ km, $L = 1000$ km, and $L = 10{,}000$ km with $S_o = 0.085$ ps/nm^2/km, $\frac{d\tau}{d\lambda} = 5 \times 10^{-3}$ ps/nm/km, $\frac{d^2\tau}{d\omega^2} = 0.138$ ps^3/km, and $\frac{d^3\tau}{d\omega^3} = 0.000618$ ps^4/km: (c) for F2, F3, and F4, and (d) comparison of different terms by exact method at $L = 10{,}000$ km.

and F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at a slightly higher frequency. Thus, we conclude that the approximation mentioned in approximate analysis is not valid and deviations reported for higher-order dispersion terms are caused by only the approximation involved for the second-order dispersion term. This reflects that for exact analysis there are no second-order dispersion effects, but there are certainly some third-order dispersion effects at high modulating frequencies and large propagation distances.

Mathematically it is observed that for approximate analysis (Equations (40) and (51)), the approximations are for second-order (F3) and fourth-order (F5) dispersion terms. No such approximations have been assumed for first-order (F2) and third-order dispersion terms. This is also clear from the exact expressions (Equations (53) and (54)) in the exact analysis. It is therefore quite obvious that in exact analysis up to the third-order dispersion term, there will be no second-order dispersion effects but there will definitely be some third-order dispersion terms. This indeed justifies our results for the third-order effects.

Conclusions

In this paper, we have developed a theory to investigate the influence of higher-order dispersion terms on optical communication systems using small-signal analysis. The theory developed in this article may be considered as an extension of the results reported in Crognale and Cartaxo and Morgado [12, 13], where the impact of third-order dispersion terms was neglected. We compare approximate and exact small-signal theories for analyzing the influence of the higher-order dispersion terms on dispersive optical communication systems operating near zero dispersion wavelength for linear single-mode fiber. For the approximate theory, the generalized conversion matrix has been reported that gives the transfer function of intensity and phase from the fiber input to fiber output for laser source, including the influence of any higher-order dispersion term. In addition, expressions for small-signal frequency response and RIN response of an ultrafast laser diode including noises are obtained. The results of the approximate theory agree with the results reported in Crognale [12], but the approximation used does not seem to be valid as reported earlier [13]. Therefore from the approximate theory, the exact conversion matrix and exact expressions for small-signal frequency response and RIN are obtained by substituting the approximation back. The deviations in approximate theory are caused by only the approximation involved. We show that for the exact theory, the second-order dispersion effects are irrelevant for the intensity and frequency response even at large modulating frequencies and large propagation distances contrary to the approximate theory as reported earlier [13]. But we show that the third-order dispersion term certainly has some impact on the frequency and RIN response for long distance links at high modulating frequencies.

References

1. Mears, R. J., L. Reekie, I. M. Jauncey, and D. N. Payne. 1987. Low noise erbium-doped fiber amplifier operating at 1.54 μm. *Electron. Lett.*, 23:1026–1027.
2. Desurvire, E., J. R. Simpson, and P. C. Becker. 1987. High gain erbium-doped travelling wave fiber amplifier. *Opt. Lett.*, 12:888–890.
3. Elrefaie, A. F., R. W. Wanger, D. A. Atlas, and D. G. Daut. 1988. Chromatic dispersion limitations in coherent lightwave systems. *J. Lightwave Technol.*, 6:704–709.

4. Ishikawa, G., H. Ooi, Y. Akiyama, S. Taniguchi, and H. Nishimoto. 1996. 80-Gb/s (2× 40 Gb/s) transmission experiments over 667-km dispersion-shifted fiber using Ti:LiNbO OTDM modulator and demultiplexer. *Proc. ECOC'96*, Oslo, Norway. Postdeadline papers, 5: 37–40.
5. Schlafer, J., C. B. Su, W. Powazinik, and R. B. Lauer. 1985. 20 GHz bandwidth InGaAs photodetector for microwave optical transmission. *Electron. Lett.*, 21:469–470.
6. Koyama, K., and Y. Suematsu. 1985. Analysis of dynamic spectral width of dynamic single mode (DSM) laser and related transmission bandwidth of single-mode fiber. *IEEE J. Quantum Electron.*, QE-21(4):292–297.
7. Agrawal, G. P. 1989. *Nonlinear Fiber Optics*. Academic Press.
8. Hasegawa, A., and Y. Kodama. 1995. *Solitons in Optical Communications*. Oxford, UK: Clarendon.
9. Agrawal, G. P., and M. J. Potasek. 1986. Nonlinear pulse distortion in single mode optical fibers at the zero dispersion wavelength in an optical fiber. *Phys. Rev. A*, 33(3):1765–1776.
10. Schultz, J., W. Hodel, and H. P. Weber. 1993. Nonlinear pulse distortion at the zero dispersion wavelength in an optical fiber. *Opt. Commun.*, 95(4/5/6):357–366.
11. Wang, J., and K. Petermann. 1992. Small-signal analysis for dispersive optical fiber communication systems. *IEEE J. Lightwave Technol.*, 10(1):96–100.
12. Crognale, C. 1997. Small-signal frequency response of a linear dispersive single-mode fiber near zero first-order dispersion wavelength. *IEEE J. Lightwave Technol.*, 15(3):482–489.
13. Cartaxo, A. V. T., and J. A. P. Morgado. 1999. Rigorous assessment of small signal analysis for linear and dispersive optical communication systems operating near zero dispersion wavelength. *IEEE J. Lightwave Technol.*, 17(1):86–94.
14. Petermann, K. 1988. *Laser Diode Modulation and Noise*. The Netherlands: Kluwer Academic.
15. Djupsjobaka, A., and O. Sahlen. 1994. Despersion compensation by differential time delay. *IEEE J. Lightwave Technol.*, 12(10):1849–1853.
16. Sharma, A. K., R. K. Sinha, and R. A. Agarwala. 1997. Improved analysis of dispersion compensation using differential time delay for high-speed long-span optical link. *Fiber and Integrated Optics*, 16(4):415–426.
17. Sharma, A. K., R. K. Sinha, and R. A. Agarwala. 1998. Higher-order dispersion compensation by differential time delay. *Optical Fiber Technology*, 4(1):135–143.
18. Cohen, L. G. 1988. Comparison of single-mode fiber dispersion measurement techniques. *IEEE J. Lightwave Technol.* LT-3(5):557–568.
19. Petermann, K. 1990. FM-AM noise conversion in dispersive single-mode fiber transmission lines. *IEE Electron Lett.*, 26(25):2097–2098.
20. Koch, T. L., and R. A. Linke. 1986. Effect of nonlinear gain reduction on semiconductor laser wavelength chirping. *Appl. Phys. Lett.*, 48(1):613–615.
21. Yamamoto, S., N. Edagawa, H. Taga, Y. Yoshida, and H. Wakabayashi. 1990. Analysis of laser phase noise to intensity noise conversion by chromatic dispersion in intensity modulation and direct detection optical-fiber transmission. *J. Lightwave Technol.*, 8(11):1716–1722.
22. ITU-T, Rec. G.653. 1992. Characteristic of dispersion shifted single mode optical fiber cable. 6.

Biographies

R. S. Kaler was born in Kausoli, Himachal Pradesh, India on 31 December 1968. He obtained his bachelor's degree in Electronics and Communication Engineering with distinction from the Department of Electronics Technology, Guru Nanak Dev University Amritsar, India. Further he obtained his Master's degree in Electronics Engineering from Punjab University, Chandigarh, India. He worked in Punjab Communication Limited, Mohali and Electronics Systems Punjab Limited Mohali from 1990 to 1994. He then joined BBSEC Patehgarh Sahib as lecturer and became Assistant Professor in 1998. He then joined Sant Longowal Institute of Engineering and Technology, Longowal, Punjab, India as Assistant Professor in the Department of Electronics and Communication Engineering in 1999. Since then, he is working for his PhD degree in the field of Optical Communication from Punjab Technical University, Jalandhar, Punjab, India. His present interests are fiber dispersion and nonlinearities. He has over thirty research papers in international/national journals/conferences. He is life member of Institution of Engineers (India) and Indian Society of Technical Education.

Dr. T. S. Kamal was born in Dhanaula, Distt. Sangrur, Punjab, India in August 1941. He obtained his bachelor's and master's degree (Gold Medallist) in Electronics and Communication Engineering from University of Roorkee, Roorkee, India. Further he obtained his doctorate's degree in Electronics and Communication Engineering from Punjab University, Chandigarh. He is Ex-Vice President, Institution of Engineers, India and presently working as Professor and Head of the Department of Electronics and Communication Engineering at Sant Longowal Institute of Engineering and Technology, Longowal, India. He has guided over nine PhD students and has over 35 years teaching experience at Punjab Engineering College, Chandigarh. He has worked in various prestigious positions at national and international levels. He has organized several national and international conferences. He has also 110 papers in international/national journals/conferences. His present interests are optical communication, wireless communications, and neural networks. He belongs to the Fellow Institution of Engineers (India), the Fellow Institute of Electronics and Telecommunication Engineers, and is a Senior Member of the IEEE (USA).

Ajay K. Sharma received his BE degree in Electronics and Electrical Communication Engineering from Punjab University, Chandigarh, India in 1986. He received his MS in Electronics and Control Engineering from Birla Institute of Technology and Science. Pilani in 1984 and his PhD in Electronics, Communication, and Computer Engineering from Kurukshetra University, Kurukshotra, India in 1999. His PhD dissertation was on "Studies of Broadband Optical Communication Systems and Networks." From 1986–1990, he was with Technical Teacher Training Institute and DTE, Chandigarh, Indian Railways New Delhi, Sant Longowal Institute of Engineering and Technology, Longowal at various positions and was responsible teaching and research in the field of Electronics Circuits and Telecommunication links. He joined Regional Engineering College, Hamirpur (HP) in 1991 where he has worked on the faculty of Electronics and Communication Engineering and was involved in teaching R & D in the field of Electronics circuits and Broad Band Optical Communication Systems and Networks. He worked as Assistant Professor from 1996–2001 at Regional Engineering College, Jalandhar, and since November 2001, he is working as Professor in the same department. He is responsible for teaching, department development, and research in the field of dispersion compensation and WDM systems. He has been involved in various sponsored and R & D projects in the field of optical communication systems and networks. He has authored nine books. He has more than 40 research papers published/presented in international/national journals/conferences to his credit. His current interests include dispersion compensation for linear and nonlinear optical communication systems, soliton transmission, and WDM Networks. He is acting as technical reviewer for the Journal of SPIE—The International Society for Optical Engineering, USA. He is also life member of the Indian Society of Technical Education (ISTE).