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Power penalty analysis for realistic weight functions using differential time delay with higher-order dispersion

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Abstract

In this paper, the power penalty analysis for approximate and realistic weight functions has been presented for combating the pulse broadening effects of group-velocity dispersion in a fiber-optic communication link using differential time delay method including higher-order dispersion terms. The expressions for root mean square (RMS) phase deviation, optimum chirp factor and figure of merit have been evaluated for approximate and realistic systems. We show that the optimum value of chirp factor corresponds to dispersion compensation. The power penalty graphs for second-, third-, and fourth-order dispersion and their combinations have been presented for distance up to 300 km for this chirp factor for different weight functions. It is observed that the power penalty for realistic weight functions is less in comparison with the approximated weight function. It has also been shown that it is possible for a short pulse to propagate without significant broadening over the lengths many times longer than the usual dispersion length of fiber.

Keywords: Compensating device; Dispersion; Figure of merit; Group-velocity dispersion; RMS phase deviation; Higher-order dispersion

1. Introduction

Recently, there has been great interest in using single mode fibers for high-bit-rate transmission in low-loss-transmission windows but dispersion is an important impairment that degrades overall system performance of an optical communication system. At high-bit rate, the dispersion-induced broadening of short pulses propagating in the fiber causes crosstalk between the adjacent time slots, leading to errors when the communication distance increases beyond the dispersion length of the fiber. Higher-order dispersion terms

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are the forces destructive of pulse propagation in ultra high-bit rate optical transmission system and cause power penalty in the system. The power penalty in presence of impairment is defined as the increase in signal power required (in dB) to maintain the same bit error rate as in the absence of that impairment. Therefore, in order to realize the high data rates over long distances down the SM fiber, techniques must be found to overcome the pulse spreading and reduce power penalty due to dispersion. Various methods for dispersion compensation have been reported in [1–14].

The work on dispersion compensation by differential time delay was first reported in [14]. The proposed scheme was intended for high-bit rate (> 10 Gb/s) time division multiplexed transmission, and it was shown that the transmitting distance could be enhanced by a factor of 4 in an approximate case and in the order of 2.5 in realistic conditions in dispersive limited system. In this technique, the analysis and implementation was based on the individual second-order (2OD) term of the propagation constant. Further, it was proved that it was superior to other existing methods.

Earlier, an attempt was made by us to compensate dispersion using above technique by using other higher-order dispersion terms [15] using unity weight function. Paper presented the improved analysis of dispersion compensation using third- and fourth-order (3OD and 4OD) dispersion terms individually. The parameters such as RMS phase deviation, figure of merit, and dimension free chirp parameter were evaluated and analyzed for this approximate optical systems for 2OD, 3OD, and 4OD.

Also we had reported our work on dispersion compensation by differential time delay using higher-order terms selectively [16] again with unity weight function. Combined effects of higher-order terms (2OD + 3OD) and (3OD + 4OD) had been analyzed when they are considered together instead of individual terms as reported in [14,15]. Further, we had again reported work on dispersion compensation by same method using higher-order terms together [17] for the same weight function. Combined effects of higher-order terms (2OD + 3OD + 4OD) had been analyzed. Again, the same parameters were evaluated for combined terms. The power penalty analysis was not reported in our earlier work [15–17] for higher-order dispersion terms.

The unity weight function means that we would require a sinc as the pulse envelope in the system. This corresponds to the approximate case and should have high power penalty as compared to realistic systems. Therefore, it is very important to analyze the results for the realistic (weight functions) optical communication systems.

In this paper, we extend our work reported in [15–17] and derive the expression for power penalty and calculate it for approximate and different realistic weight functions. We show that the figure of merit decreases for realistic optical communication systems. We also show that for zero value of chirp factor, the power penalty is minimum. The Section 2 deals with the theory of differential delay dispersion compensation method and derivation of expression for power penalty including higher-order dispersion terms. In Section 3, we first reproduce the approximate case [17] for deriving expression for RMS phase deviation, dimension free chirp parameter and figure of merit for approximate optical communication systems by considering all higher-order dispersion terms together for unit weight function. We then derive same expressions for three more realistic weight functions. Based on similar derivations, we report all our results [15–17] in the form of table. We then in Section 4 use these results (chirp parameter) to plot power penalty for all the different cases of individual dispersion terms and their combinations for different weight functions. The comparisons in power penalty are made and conclusions are drawn.

2. Power penalty analysis including higher-order dispersion

We begin with the fundamental observation that dispersion in an optical fiber is a linear process when power levels are kept below those required the onset of self-phase modulation or stimulated Raman scattering or nonlinear effects. Thus, the transmission characteristics of a single mode fiber can be represented by a linear filter, which can be derived by Taylor expansion of the propagation constant β about the optical carrier frequency ω_0 as mentioned in [14,15]

$$\beta = \beta_0 + \frac{1}{v_g}(\omega - \omega_0) - \frac{\lambda^2 D}{4\pi c}(\omega - \omega_0)^2 + \frac{1}{6}(\omega - \omega_0)^3 \frac{d^3\beta}{d\omega^3} + \frac{1}{24}(\omega - \omega_0)^4 \frac{d^4\beta}{d\omega^4} + \cdots, \quad (1)$$

where $v_g = d\omega/d\beta$ is group velocity and D is standard group delay dispersion parameter given by

$$D = \frac{\partial}{\partial\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\frac{\partial^2\beta}{\partial\omega^2}. \quad (2)$$

For simplicity, the propagation constant can be expressed as

$$\beta = \beta_0 + (\Delta\omega)D_1 + \tfrac{1}{2}(\Delta\omega)^2 D_2 + \tfrac{1}{6}(\Delta\omega)^3 D_3 + \tfrac{1}{24}(\Delta\omega)^4 D_4 + \cdots, \quad (3)$$

where

$$D_1 = \frac{\partial\beta}{\partial\omega}, \qquad D_2 = \frac{\partial^2\beta}{\partial\omega^2}, \qquad D_3 = \frac{\partial^3\beta}{\partial\omega^3}, \qquad D_4 = \frac{\partial^4\beta}{\partial\omega^4} \quad (4)$$

are first-, second-, third-, and fourth-order dispersion terms, respectively.

At the receiver, the phase deviation may be expressed as

$$\phi = \phi_0 - \beta L, \quad (5)$$

where L is transmission distance of the fiber.

Therefore

$$\phi = \phi_0 - \beta_0 L - (\Delta\omega)D_1 L - \tfrac{1}{2}(\Delta\omega)^2 D_2 L - \tfrac{1}{6}(\Delta\omega)^3 D_3 L - \tfrac{1}{24}(\Delta\omega)^4 D_4 L - \cdots. \quad (6)$$

First two terms on right-hand side of Eq. (6) are constant phase contributions and third term gives pure time delay. In the analysis the demands of absolute phase and time has been neglected. The phase contributions offered by the higher-order terms have been analyzed for different cases as discussed in [15–17].

The calculation of RMS value of phase deviation is the most common way to know the degradation in the dispersive systems. The phase deviation for the compensating device can be expressed as

$$\phi_{n\mathrm{RMS}} = \left[\int_{\omega-\omega_0}^{\omega+\omega_0} \frac{P(\Delta\omega)\phi_n(\mathrm{total})^2\, d\omega}{2\omega_0}\right]^{1/2}, \quad (7)$$

where $n = 1, 2, 3, 4$ correspond to the dispersion terms and $P(\omega)$ is the dimension free weight function.

The deviation in time for arrival of frequencies can be expressed as

$$\Delta t = t - t_0 = -\frac{d\phi_n}{d\omega}, \tag{8}$$

where t_0 is the arrival for the carrier frequency.

In general, the dispersion compensation is based on spectral separation of the signal in upper and lower sidebands and after providing differential time delay they are synchronously combined together for further transmission and detection. The upper and lower sidebands are separated through an integrated interferometer and time delay is provided in each arm.

We know derive expression for power penalty for dispersive optical communication systems by including higher-order dispersion terms. The propagation equation comprising of dispersion terms up to the fourth-order can be written as [18]

$$\frac{\partial A}{\partial z} + D_1 \frac{\partial A}{\partial t} + \frac{iD_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{1}{6} D_3 \frac{\partial^3 A}{\partial t^3} + \frac{iD_4}{24}\frac{\partial^4 A}{\partial t^4} = 0. \tag{9}$$

We consider propagation of Gaussian pulses in the optical fibers by considering the initial amplitude as

$$A(0,t) = A_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^2\right], \tag{10}$$

where A_0 is the peak amplitude and parameter T_0 represents half width at $1/e$ intensity point and is related with full width at half maximum (FWHM) by the relation

$$T_{\mathrm{FWHM}} = 2(\ln 2)^{1/2} T_0. \tag{11}$$

The parameter C governs linear frequency chirp imposed on the chirp. A pulse is chirped if it's carrier frequency changes with time. The frequency change is expressed by Eq. (8) and is given by

$$\delta\omega = -\frac{\partial \phi}{\partial t} = \frac{C}{T_0^2} t. \tag{12}$$

The time-dependent quantity $\delta\omega$ is called the chirp. Its inclusion is important since semiconductor lasers generally emit pulses that are considerably chirped. The Fourier spectrum of chirped pulse is broader than that of the unchirped pulse. Taking Fourier transform of the initial pulse amplitude, we get

$$A(0,\omega) = A_0 \left(\frac{2\pi T_0^2}{1+iC}\right) \exp\left[-\frac{\omega^2 T_0^2}{2(1+iC)}\right]. \tag{13}$$

The pulse propagation equation can be easily solved in the Fourier domain, its solution is given by

$$A(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(0,\Delta\omega) \exp\left[iD_1 z \Delta\omega + \frac{i}{2} D_2 z \Delta\omega^2 + \frac{i}{6} D_3 z \Delta\omega^3 + \frac{i}{24} D_4 z \Delta\omega^4 - i\Delta\omega t\right] d(\Delta\omega). \tag{14}$$

Integrating the equation and finding T/T_0 at $1/e$th intensity point, we get

$$\frac{T}{T_0}=\left[\left(1+\frac{CD_2z}{T_0^2}\right)^2+\left(\frac{D_2z}{T_0^2}\right)^2+\left(1+C^2\right)\left(\frac{D_3z}{2T_0^3}\right)^2+\left(1+C^2+C^4\right)\left(\frac{D_4z}{2T_0^4}\right)^2\right]^{1/2}. \tag{15}$$

The broadening factor is defined as σ/σ_0, where σ_0 is the RMS width of the input Gaussian pulse ($\sigma_0=T_0/\sqrt{2}$) as given in [18]. Substituting in Eq. (15), we get

$$\frac{\sigma}{\sigma_0}=\left[\left(1+\frac{CD_2z}{2\sigma_0^2}\right)^2+\left(\frac{D_2z}{2\sigma_0^2}\right)^2+\left(1+C^2\right)\frac{1}{2}\left(\frac{D_3z}{4\sigma_0^3}\right)^2+\left(1+C^2+C^4\right)\left(\frac{D_4z}{8\sigma_0^4}\right)^2\right]^{1/2}. \tag{16}$$

The power penalty is given at by

$$PP=10\log_{10}\frac{\sigma}{\sigma_0}. \tag{17}$$

The RMS pulse width should be such that $4\sigma_0\leqslant 1/B$, where B is the bit rate. By choosing the worst-case condition $\sigma_0=1/4B$, the power penalty at $z=L$ is given by

$$PP=5\log_{10}\left[\left(1+8CD_2B^2L\right)^2+\left(8D_2B^2L\right)^2+\left(1+C^2\right)\tfrac{1}{2}\left(16D_3B^3L\right)^2+\left(1+C^2+C^4\right)\left(32D_4LB^4\right)^2\right], \tag{18}$$

which is the desired expression for power penalty due to higher-order dispersion terms.

3. Optimum chirp and figure of merit for realistic weight functions

In this section, we derive/reproduce expressions for optimum chirp factor and figure of merit for approximate and realistic weight functions. In Eq. (5) neglecting the demands of absolute phase and time at the receiver side and considering the 2OD, 3OD, and 4OD term, the phase deviation can be expressed as

$$\phi_{234}=\tfrac{1}{2}(\Delta\omega)^2D_2L+\tfrac{1}{6}(\Delta\omega)^3D_3L+\tfrac{1}{24}(\Delta\omega)^4D_4L. \tag{19}$$

The time delay in the arms of the interferometer can be expressed as $\pm C_{234}\omega_{\mathrm{cl}}(D_2L/2+D_3L\Delta\omega/4+D_4L\Delta\omega^2/12)$, where C_{234} is the dimensionless parameter and ω_{cl} is the angular clock frequency of the system. The total phase deviation can be expressed as

$$\phi_{234}(\text{total})=D_2L\left(\Delta\omega^2\pm C_{234}\omega_{\mathrm{cl}}\Delta\omega\right)/2+D_3L\left(2\Delta\omega^3\pm 3C_{234}\omega_{\mathrm{cl}}\Delta\omega^2\right)/12+D_4L\left(\Delta\omega^4\pm 2C_{234}\omega_{\mathrm{cl}}\Delta\omega^3\right)/24. \tag{20}$$

3.1. Case I

We reproduce the general case reported in [17] to evaluate the expression for RMS phase deviation, dimension free chirp parameter and figure of merit for approximate optical communication systems using higher-order dispersion terms together.

Substituting Eq. (20) in Eq. (7) for $n = 234$ and $P(\omega) = 1 = P(1)$

$$\phi_{234\mathrm{RMS}} = \frac{L}{24\sqrt{\omega_{\mathrm{cl}}}}\left[\int_0^{\omega_{\mathrm{cl}}} \left\{12D_2\left(\omega^2 - C_{234}\omega_{\mathrm{cl}}\omega\right) + 2D_3\left(2\omega^3 - 3C_{234}\omega_{\mathrm{cl}}\omega^2\right) + D_4\left(\omega^4 - 2C_{234}\omega_{\mathrm{cl}}\omega^3\right)\right\}^2 \mathrm{d}\omega\right]^{1/2}. \quad (21)$$

At $\omega = \omega_{\mathrm{cl}}$, $\phi_{234\mathrm{RMS}}$ is found to be

$$\begin{aligned}\phi_{234\mathrm{RMS}} = \frac{L}{24\sqrt{\omega_{\mathrm{cl}}}}\bigg[&144D_2^2\omega_{\mathrm{cl}}^5\left(\frac{1}{5} - \frac{C_{234}}{2} + \frac{C_{234}^2}{3}\right)\\ &+ 4D_3^2\omega_{\mathrm{cl}}^7\left(\frac{4}{7} - 2C_{234} + \frac{9C_{234}^2}{5}\right) + 4D_4^2\omega_{\mathrm{cl}}^9\left(\frac{1}{9} - \frac{C_{234}}{2} + \frac{4C_{234}^2}{7}\right)\\ &+ 48D_2D_3\omega_{\mathrm{cl}}^6\left(\frac{1}{3} - C_{234} + \frac{3C_{234}^2}{4}\right) + 24D_2D_4\omega_{\mathrm{cl}}^7\left(\frac{1}{7} - \frac{C_{234}}{2} + \frac{2C_{234}^2}{5}\right)\\ &+ 4D_3D_4\omega_{\mathrm{cl}}^8\left(\frac{1}{4} - C_{234} - C_{234}^2\right)\bigg]^{1/2}. \end{aligned} \quad (22)$$

Hence at $\omega = \omega_{\mathrm{cl}}$ in terms of D_4, $D_2 = \omega_{\mathrm{cl}}^2 D_4/2$, and $D_3 = \omega D_4$. Eq. (22) reduces to

$$\begin{aligned}\phi_{234\mathrm{RMS}} = \frac{D_4\omega_{\mathrm{cl}}^4 L}{24}\bigg[&36\left(\frac{1}{5} - \frac{C_{234}}{2} + \frac{C_{234}^2}{3}\right) + 4\left(\frac{4}{7} - 2C_{234} + \frac{9C_{234}^2}{5}\right)\\ &+ 24\left(\frac{1}{3} - C_{234} + \frac{3C_{234}^2}{4}\right) + \left(\frac{1}{9} - \frac{C_{234}}{2} + \frac{4C_{234}^2}{7}\right)\\ &+ 12\left(\frac{1}{7} - \frac{C_{234}}{2} + \frac{2C_{234}^2}{5}\right) + 4\left(\frac{1}{4} - C_{234} - C_{234}^2\right)\bigg]^{1/2}, \end{aligned} \quad (23)$$

$$\phi_{234\mathrm{RMS}} = \frac{D_4\omega_{\mathrm{cl}}^4 L}{24}\left[20.311 - 60.5C_{234} + 46.57C_{234}^2\right]^{1/2}, \quad (24)$$

$$\phi_{234\mathrm{RMS}} = \frac{D_4 L\omega_{\mathrm{cl}}^4}{24}\sqrt{\alpha}, \quad (25)$$

where

$$\alpha = 20.311 - 60.5C_{234} + 46.57C_{234}^2. \quad (26)$$

To minimize the RMS value of phase deviation, putting $\mathrm{d}\alpha/\mathrm{d}C_{234} = 0$, it follows that $C_{234} = 0.65$. Substituting in Eq. (24) to give the optimum value

$$\phi_{234\mathrm{RMS}}(\mathrm{opt}) = \frac{D_4 L\omega_{\mathrm{cl}}^4}{2}\sqrt{0.6618}. \quad (27)$$

Further with $C_{234} = 0$, we get

$$\phi_{234\mathrm{RMS}}(C_{234} = 0) = \frac{D_4 L \omega_{\mathrm{cl}}^4}{2} \sqrt{20.311}. \tag{28}$$

The figure of merit for dispersion compensating device can be found from Eqs. (27) and (28)

$$G_{234} = \frac{\phi_{234\mathrm{RMS}}(C_{234} = 0)}{\phi_{2342\mathrm{RMS}}(\mathrm{opt})} = 5.54. \tag{29}$$

3.2. Case II

We now derive the expression for optimum chirp factor and figure of merit using the same method but taking realistic weight function $P(\omega) = 1 - |\omega/\omega_{\mathrm{cl}}| = P(2)$. Substituting Eq. (20) in Eq. (7) for $n = 234$ and $P(2)$, we get

$$\phi_{234\mathrm{RMS}} = \frac{L}{24\sqrt{\omega_{\mathrm{cl}}}} \left[\int_0^{\omega_{\mathrm{cl}}} \left(1 - \left|\frac{\omega}{\omega_{\mathrm{cl}}}\right|\right) \left\{12 D_2\left(\omega^2 - C_{234}\omega_{\mathrm{cl}}\omega\right) + 2D_3\left(2\omega^3 - 3C_{234}\omega_{\mathrm{cl}}\omega^2\right) + D_4\left(\omega^4 - 2C_{234}\omega_{\mathrm{cl}}\omega^3\right)\right\}^2 \mathrm{d}\omega \right]^{1/2}. \tag{30}$$

Hence at $\omega = \omega_{\mathrm{cl}}$ in terms of D_4, $D_2 = \omega_{\mathrm{cl}}^2 D_4/2$, and $D_3 = \omega D_4$. Eq. (30) reduces to

$$\phi_{234\mathrm{RMS}} = \frac{D_4 \omega_{\mathrm{cl}}^4 L}{24} \left[2.965 - 10.155 C_{234} + 9.243 C_{234}^2\right]^{1/2}, \tag{31}$$

$$\phi_{234\mathrm{RMS}} = \frac{D_4 L \omega_{\mathrm{cl}}^4}{24} \sqrt{\alpha}, \tag{32}$$

where

$$\alpha = 2.965 - 10.155 C_{234} + 9.243 C_{234}^2. \tag{33}$$

To minimize the RMS value of phase deviation, putting $\mathrm{d}\alpha/\mathrm{d}C_{234} = 0$, it follows that $C_{234} = 0.55$. Substituting in Eq. (32) to give the optimum value

$$\phi_{234\mathrm{RMS}}(\mathrm{opt}) = \frac{D_4 L \omega_{\mathrm{cl}}^4}{2} \sqrt{0.175}. \tag{34}$$

Further with $C_{234} = 0$, we get

$$\phi_{234\mathrm{RMS}}(C_{234} = 0) = \frac{D_4 L \omega_{\mathrm{cl}}^4}{2} \sqrt{2.965}. \tag{35}$$

The figure of merit case for dispersion compensating device can be found from Eqs. (34) and (35)

$$G_{234} = \frac{\phi_{234\mathrm{RMS}}(C_{234} = 0)}{\phi_{2342\mathrm{RMS}}(\mathrm{opt})} = 4.11. \tag{36}$$

3.3. Case III

We now derive the optimum chirp factor and figure of merit using the same method but taking realistic weight function $P(\omega) = \cos^2(\pi\omega/2\omega_{cl}) = P(3)$.

Substituting Eq. (20) in Eq. (7) for $n = 234$ and $P(3)$, we get

$$\phi_{234\mathrm{RMS}} = \frac{L}{24\sqrt{\omega_{cl}}}\left[\int_0^{\omega_{cl}} \cos^2\left(\frac{\pi\omega}{2\omega_{cl}}\right)\left\{12D_2\left(\omega^2 - C_{234}\omega_{cl}\omega\right) + 2D_3\left(2\omega^3 - 3C_{234}\omega_{cl}\omega^2\right) + D_4\left(\omega^4 - 2C_{234}\omega_{cl}\omega^3\right)\right\}^2 \mathrm{d}\omega\right]^{1/2}. \tag{37}$$

Hence in terms of D_4, $D_2 = \omega_{cl}^2 D_4/2$, and $D_3 = \omega D_4$. Eq. (37) reduces to

$$\phi_{234\mathrm{RMS}} = \frac{LD_4}{24\sqrt{2\omega_{cl}}}\left[\int_0^{\omega_{cl}} \left(I_1'' + I_2''\right)\mathrm{d}\omega\right]^{1/2}, \tag{38}$$

where

$$I_1'' = \int_0^{\omega_{cl}} \Big[\omega^8 + \omega^7\omega_{cl}(8 - 4C_{234}) + \omega^6\omega_{cl}^2\left(28 - 28C_{234} + 4C_{234}^2\right) + \omega^5\omega_{cl}^3\left(48 + 96C_{234} - 24C_{234}^2\right) + \omega^4\omega_{cl}^4\left(36 - 120C_{234} + 60C_{234}^2\right) + \omega^3\omega_{cl}^5\left(36\omega^6\omega_{cl}^2 - 72C_{234} + 72C_{234}^2\right)\Big]\mathrm{d}\omega, \tag{39}$$

$$I_2'' = \int_0^{\omega_{cl}} \cos\left(\frac{\pi\omega}{\omega_{cl}}\right) I_1''\,\mathrm{d}\omega. \tag{40}$$

Substituting the values at $\omega = \omega_{cl}$

$$\phi_{234\mathrm{RMS}} = \frac{D_4\omega_{cl}^4 L}{24}\left[3.35917 - 12.685C_{234} + 12.86C_{234}^2\right]^{1/2}, \tag{41}$$

$$\phi_{234\mathrm{RMS}} = \frac{D_4 L\omega_{cl}^4}{24}\sqrt{\alpha}, \tag{42}$$

where

$$\alpha = 3.35917 - 12.685C_{234} + 12.86C_{234}^2. \tag{43}$$

To minimize the RMS value of phase deviation, putting $\mathrm{d}\alpha/\mathrm{d}C_{234} = 0$, it follows that $C_{234} = 0.49$. Substituting in Eq. (42) to give the optimum value

$$\phi_{234\mathrm{RMS}}(\mathrm{opt}) = \frac{D_4 L\omega_{cl}^4}{2}\sqrt{0.23}. \tag{44}$$

Further with $C_{234} = 0$, we get

$$\phi_{234\mathrm{RMS}}(C_{234} = 0) = \frac{D_4 L\omega_{cl}^4}{2}\sqrt{3.35917}. \tag{45}$$

The figure of merit for dispersion compensating device can be found from Eqs. (44) and (45)

$$G_{234} = \frac{\phi_{234\mathrm{RMS}}(C_{234}=0)}{\phi_{2342\mathrm{RMS}}(\mathrm{opt})} = 3.82. \tag{46}$$

3.4. Case IV

Taking realistic weight function

$$P(\omega) = \frac{10}{9}\cos^2\left(\frac{\pi\omega}{2\omega_{\mathrm{cl}}}\right) - \frac{1}{9}\cos^2\left(\frac{3\pi\omega}{2\omega_{\mathrm{cl}}}\right) = P(4).$$

Substituting Eq. (20) in Eq. (7) for $n = 234$ and $P(4)$, we get

$$\phi_{234\mathrm{RMS}} = \frac{L}{24\sqrt{\omega_{\mathrm{cl}}}}\left[\int_0^{\omega_{\mathrm{cl}}}\left(\frac{10}{9}\cos^2\left(\frac{\pi\omega}{2\omega_{\mathrm{cl}}}\right) - \frac{1}{9}\cos^2\left(\frac{3\pi\omega}{2\omega_{\mathrm{cl}}}\right)\right) \times \left\{12D_2\left(\omega^2 - C_{234}\omega_{\mathrm{cl}}\omega\right) + 2D_3\left(2\omega^3 - 3C_{234}\omega_{\mathrm{cl}}\omega^2\right) + D_4\left(\omega^4 - 2C_{234}\omega_{\mathrm{cl}}\omega^3\right)\right\}^2 \mathrm{d}\omega\right]^{1/2}. \tag{47}$$

Hence in terms of D_4, $D_2 = \omega_{\mathrm{cl}}^2 D_4/2$, and $D_3 = \omega D_4$. Eq. (47) reduces to

$$\phi_{234\mathrm{RMS}} = \frac{LD_4}{24\sqrt{2\omega_{\mathrm{cl}}}}\left[\int_0^{\omega_{\mathrm{cl}}}\left(I_1'' + I_2'' + I_3'''\right)\mathrm{d}\omega\right]^{1/2}, \tag{48}$$

where

$$I_1'' = \int_0^{\omega_{\mathrm{cl}}}\Big[\omega^8 + \omega^7\omega_{\mathrm{cl}}(8 - 4C_{234}) + \omega^6\omega_{\mathrm{cl}}^2\left(28 - 28C_{234} + 4C_{234}^2\right) + \omega^5\omega_{\mathrm{cl}}^3\left(48 + 96C_{234} - 24C_{234}^2\right) + \omega^4\omega_{\mathrm{cl}}^4\left(36 - 120C_{234} + 60C_{234}^2\right) + \omega^3\omega_{\mathrm{cl}}^5\left(36\omega^6\omega_{\mathrm{cl}}^2 - 72C_{234} + 72C_{234}^2\right)\Big]\mathrm{d}\omega, \tag{49}$$

$$I_2'' = \int_0^{\omega_{\mathrm{cl}}}\cos\left(\frac{\pi\omega}{\omega_{\mathrm{cl}}}\right) I_1''\,\mathrm{d}\omega, \quad \text{and} \tag{50}$$

$$I_3'' = \int_0^{\omega_{\mathrm{cl}}}\cos\left(\frac{3\pi\omega}{\omega_{\mathrm{cl}}}\right) I_1''\,\mathrm{d}\omega. \tag{51}$$

Substituting the values at $\omega = \omega_{\mathrm{cl}}$

$$\phi_{234\mathrm{RMS}} = \frac{D_4\omega_{\mathrm{cl}}^4 L}{24}\left[2.127 - 8.639C_{234} + 9.897C_{234}^2\right]^{1/2}, \tag{52}$$

$$\phi_{234\mathrm{RMS}} = \frac{D_4 L\omega_{\mathrm{cl}}^4}{24}\sqrt{\alpha}, \tag{53}$$

where

$$\alpha = 2.127 - 8.639C_{234} + 9.897C_{234}^2. \tag{54}$$

To minimize the RMS value of phase deviation, putting $d\alpha/dC_{234} = 0$, it follows that $C_{234} = 0.45$. Substituting in Eq. (53) to give the optimum value

$$\phi_{234\mathrm{RMS}}(\mathrm{opt}) = \frac{D_4 L\omega_{\mathrm{cl}}^4}{2}\sqrt{0.24}. \tag{55}$$

Further with $C_{234} = 0$, we get

$$\phi_{234\mathrm{RMS}}(C_{234} = 0) = \frac{D_4 L\omega_{\mathrm{cl}}^4}{2}\sqrt{2.127}. \tag{56}$$

The figure of merit for dispersion compensating device can be found from Eqs. (55) and (56)

$$G_{234} = \frac{\phi_{234\mathrm{RMS}}(C_{234} = 0)}{\phi_{2342\mathrm{RMS}}(\mathrm{opt})} = 2.976. \tag{57}$$

Similarly, the RMS phase deviation, dimension free chirp parameter and figure of merit for independent dispersion terms and their other combinations using same realistic weight functions can be obtained.

4. Results and discussion

The comparison of various values of optimum chirp (C) and figure of merit (G) for the approximate and realistic weight functions for independent dispersion terms and their combinations is shown in Table 1. For the realistic weight functions, the C value is normally within the interval [0.6–0.48], [0.48–0.39], and [0.39–0.32] and G value is normally within the interval [3.2–3.0], [4.6–4.2], and [6.1–5.9] if obtained for 2OD, 3OD, and 4OD terms individually and respectively. Similarly, for realistic systems, the C value is

Table 1

$P(\omega)$	2ODT		3ODT		4ODT	
	C	G	C	G	C	G
$P(1) = 1$ (approximate)	0.75	4.0	0.55	6.0	0.43	8.0
$P(2) = 1 - \lvert\omega/\omega_0\rvert$ (realistic case I)	0.60	3.2	0.48	4.6	0.39	6.1
$P(3) = \cos^2(\pi\omega/2\omega_0)$ (realistic case II)	0.53	3.0	0.43	4.3	0.35	5.9
$P(4) = 10/9\cos^2(\pi\omega/2\omega_0) - 1/9\cos^2(3\pi\omega/2\omega_0)$ (realistic case III)	0.48	3.1	0.39	4.2	0.32	6.1
	2ODT + 3ODT		3ODT + 4ODT		2OD + 3ODT 4ODT	
	C	G	C	G	C	G
$P(1) = 1$ (approximate)	0.67	5.0	0.53	6.58	0.65	5.54
$P(2) = 1 - \lvert\omega/\omega_0\rvert$ (realistic case I)	0.56	4.02	0.46	4.96	0.55	4.11
$P(3) = \cos^2(\pi\omega/2\omega_0)$ (realistic case II)	0.50	3.52	0.42	4.54	0.49	3.81
$P(4) = 10/9\cos^2(\pi\omega/2\omega_0) - 1/9\cos^2(3\pi\omega/2\omega_0)$ (realistic case III)	0.45	3.54	0.38	4.3	0.45	2.97

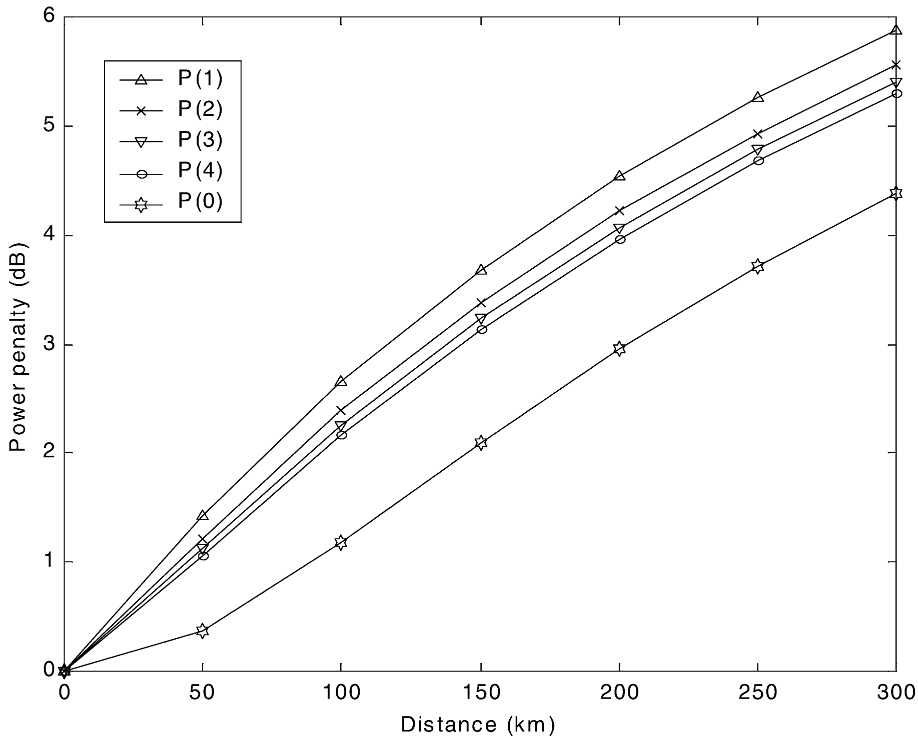

Fig. 1. Power penalty vs distance for 2OD.

normally within the interval [0.56–0.45] and [0.46–0.38], and G value is normally within the interval [4.02–3.54] and [4.96–4.30] if obtained for 2OD + 3OD and 3OD + 4OD terms together, respectively. As reported in this paper, the C value is normally within the interval [0.45–0.65] and G value is normally within the interval [3.72–5.54] if obtained for 2OD + 3OD + 4OD terms together for the realistic weight functions.

The results here show that C and G values are more accurate (because of combined effects dispersion terms) and within the values obtained in [15] using individual dispersion terms. Hence, it is possible to enhance the transmitting distance of an optical communication system (by figure of merit G for specific case) without degrading signal quality when the dispersion compensating device uses higher-order dispersion terms.

The power penalty for the different cases has been calculated as per Eq. (19) and plotted in Figs. 1–6 for optimum and zero chirp values for approximate and different realistic systems. These graphs have been plotted for dispersion of 17 ps/nm/km at 1550 nm with 10 Gb/s bit rate with $D_2 = 26.41\ \text{ps}^2/\text{km}$, $D_3 = 0.0141\ \text{ps}^3/\text{km}$, and $D_4 = 0.0068\ \text{ps}^4/\text{km}$. Each graph has been plotted for zero and optimum chirp values for approximate and realistic systems. It is observed for 300 km distance, the power penalty is varying from 0–6 dB for 2OD dispersion that is well coincident with the results reported in [14]. As is obvious, the power penalty for realistic systems is less in comparison with the approximated weight function as shown in Fig. 1. Amongst the realistic functions, $P(4)$ has least power penalty as compared to $P(3)$ and $P(2)$. $P(0)$ is the power penalty for zero chirp factor and is the minimum. $P(1)$ is the power penalty for approximate weight function of unity and is the maximum.

The power penalty for 3OD is found to be varying from 0.0–0.3 dB up to distance of 300 km. This reflects that the impact of third-order is very small as compared to the second-order dispersion. The power penalty for 4OD is varying from 0.0–0.04 dB up to

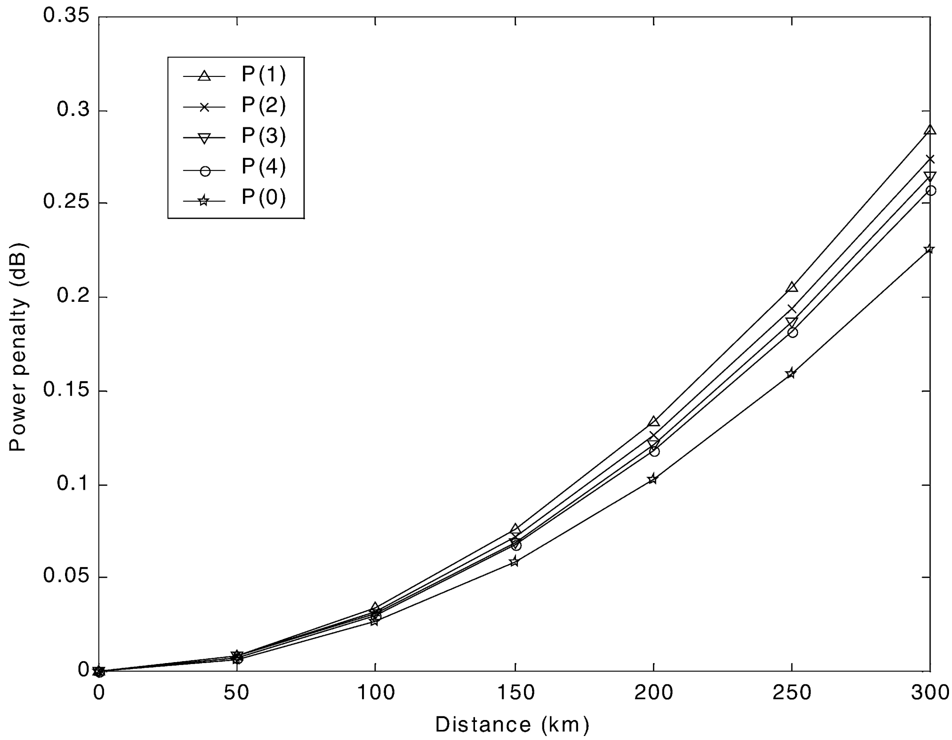

Fig. 2. Power penalty vs distance for 3OD.

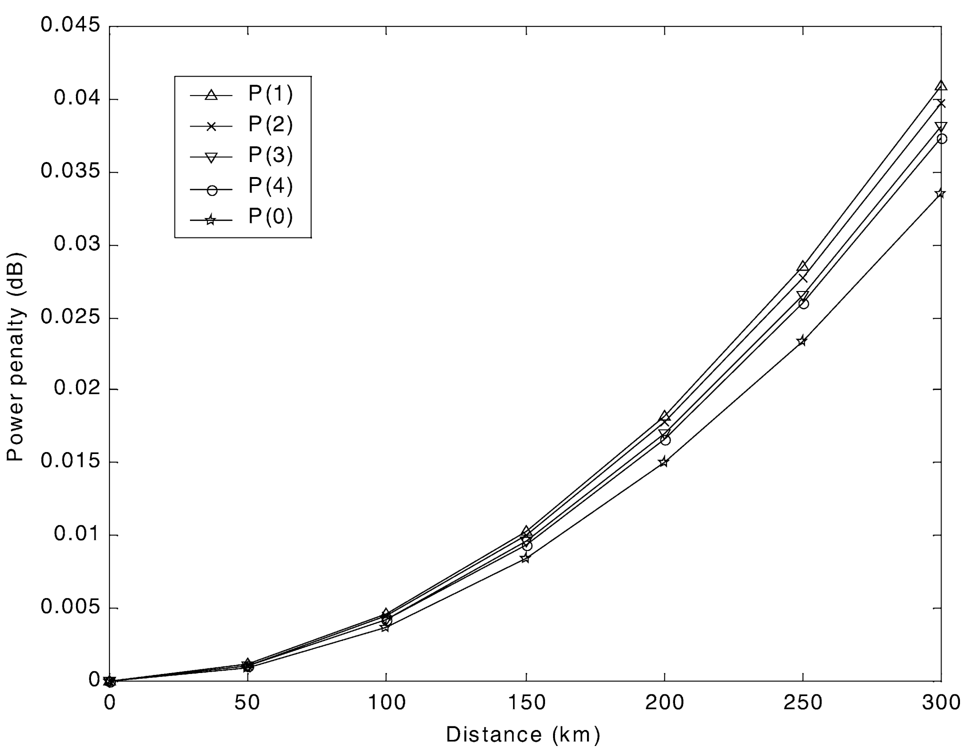

Fig. 3. Power penalty vs distance for 4OD.

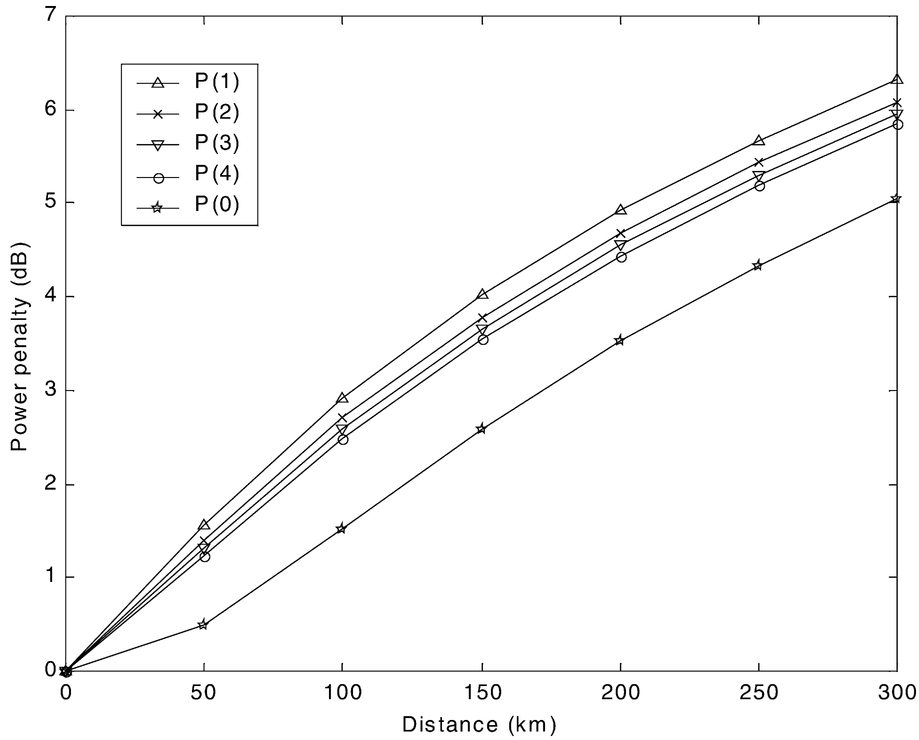

Fig. 4. Power penalty vs distance for 2OD and 3OD.

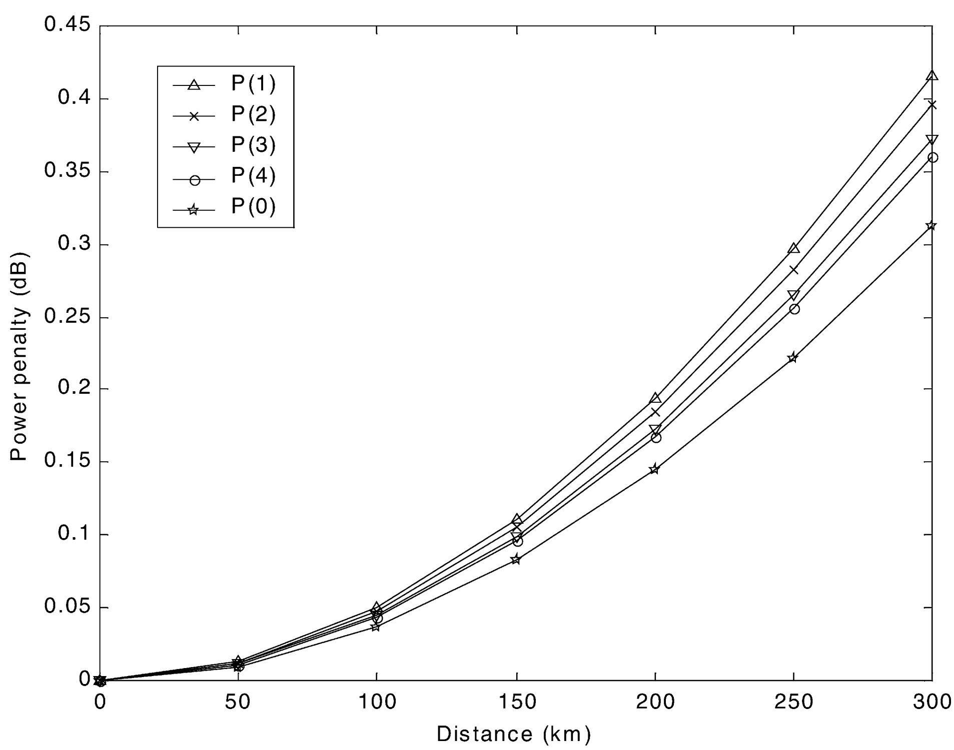

Fig. 5. Power penalty vs distance for 3OD and 4OD.

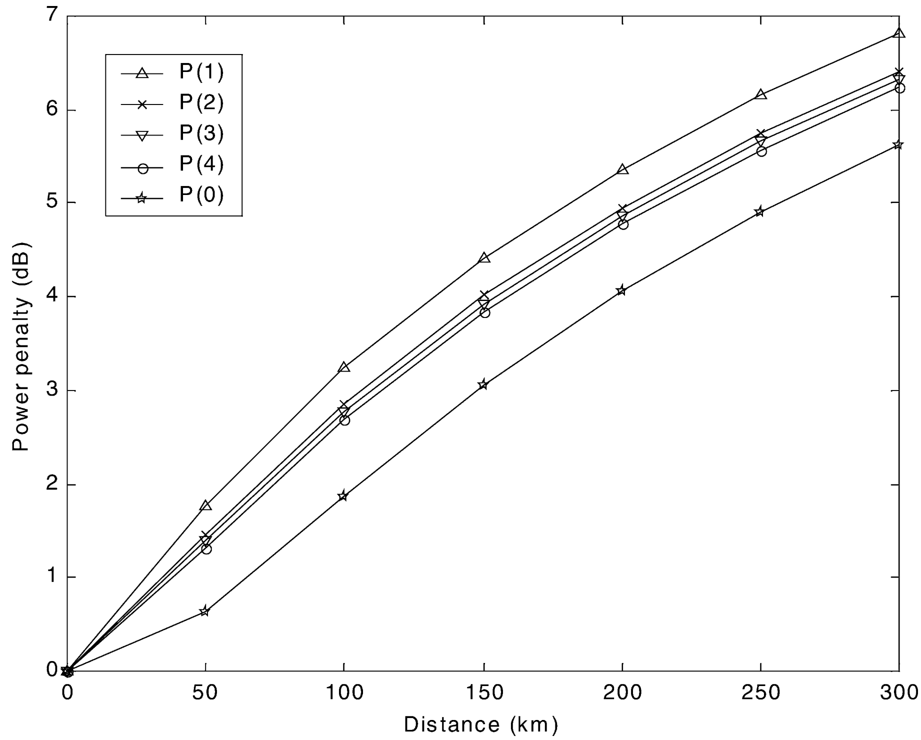

Fig. 6. Power penalty vs distance for 2OD + 3OD + 4OD.

distance of 300 km, which is further very small in comparison with second- and third-order dispersions. For combined cases of 2OD and 3OD, it is seen that the power penalty has increased slightly from the case of second-order dispersion and is varying from 0.0–6.4 dB up to distance of 300 km. For 3OD and 4OD, it has been observed that the power penalty has increased slightly from the case of third- and fourth-order dispersions individually and is varying from 0.0–0.4 dB up to distance of 300 km. For the combined case of 2OD, 3OD, and 4OD, it is found that the power penalty has increased slightly from the combined case of second- and third-order dispersions and is varying from 0.0–6.8 dB up to distance of 300 km. This gives the practical picture with all higher-order dispersion effects included. For all the cases, the power penalty for the realistic weight functions is less than the approximate weight function.

Practically, this method proposed can be implemented using an integrated interferometer as mentioned in [14,15]. The method will be most easy to implement at very high-bit rates, i.e., 20 or 40 Gb/s, since the power spectrum in that case is wider. The wider the modulated power spectrum, the easier it will be to implement low-loss, sharp, polarization-independent optical fibers [14]. The method can also be easily implemented through integrated photonic chips with laser amplifiers and detectors [14]. Several delay lines can be placed on the same chip for suitable design. The proposed method will be very helpful in choosing the design parameters for practical high-bit rate and long distance optical communication links. By employing practical dispersion compensated devices like dispersion compensated fibers and Bragg's gratings, the dispersion terms can be compensated. Then by choosing the optimum value of chirp factor based on practical realistic weight functions, the performance of optical communication systems can be optimized in terms of power penalty and transmission distance.

5. Conclusions

Paper presents the detailed theoretical power penalty analysis for the dispersion compensation by differential time delay using higher-order (2OD, 3OD, and 4OD) dispersion terms for approximate and realistic optical communication systems. It has been shown that the dispersionless propagation length can be enhanced over many times if compensation is performed using higher-order dispersion terms. For realistic optical communication systems, this propagation length decreases. The power penalty due to all the combinations of dispersion cases has been analyzed and it is observed that the higher-order dispersion terms have significant impact on the power penalty of the system. This impact decreases as the order of dispersion term increases. The impact of 3OD and 4OD is small as compared to 2OD but still has contribution when the combined terms are considered. The power penalty for realistic systems is less in comparison with the approximated weight function as these are more accurate. Amongst the realistic functions, $P(4)$ has least power penalty as compared to $P(3)$ and $P(2)$. $P(0)$ is the power penalty for zero chirp factor and is the minimum. $P(1)$ is the power penalty for approximate weight function of unity and is the maximum.

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