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# Small signal analysis with higher-order dispersion for dispersive optical communication systems

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**Abstract**

In this paper an improved small-signal analysis has been presented for analyzing the influence of the higher-order dispersion terms on dispersive optical communication systems operating near zero-dispersion wavelength for single mode fiber. A generalized conversion matrix has been reported that gives the transfer function of intensity and phase from the fiber input to fiber output for laser source including the influence of any higher-order dispersion term. In addition, analysis is applicable to evaluate the impact of combined or independent dispersion terms on small-signal frequency response and relative intensity noise (RIN) response of an ultrafast laser diode including noises due to laser spontaneous emission rate and average photon density. These responses are plotted for second-, third-, and fourth-order dispersion terms and their combinations. It is observed that the higher-order dispersion terms have significant impact on frequency and RIN responses at large modulating frequencies and large propagation distances.


*Keywords:* Dispersion; Laser noise; Optical propagation; Relative intensity noise; Higher-order dispersion

## 1. Introduction

The invention of the erbium-doped fiber amplifier (EDFA) [1,2] paved the way for the development of high bit rate, all optical ultra long-distance communicationsystems. Specifically, periodic compensation of fiber loss by EDFAs eliminates the need for electronic repeaters along the transmission line and enables the construction of all-optical communication systems in which the transmission distance is limited by the fiber chromatic dispersion rather by the fiber loss [3] because it introduces signal distortion and noise [4–8]. However, if conventional 1.3 μm zero dispersion optical fiber systems and networks are used for the 1.55 μm signal light, they exhibit a significant dispersion yielding, e.g., limitations with respect to transmission bandwidth [9,10].

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Wang et al. [11] developed a new approach to investigate the influence of the dispersion on optical fiber communication systems using small-signal analysis near zero dispersion wavelength. The conversion between phase and intensity modulation or noise caused by chromatic dispersion had been analyzed for laser diodes. A conversion matrix describing the transfer function of intensity and frequency modulation at fiber input to the intensity and frequency modulation at fiber output was reported and the results were obtained to analyze the performance of optical communication systems.

A theory describing the propagation of signal and noise through a lossless linear dispersive single mode fiber with second-order dispersion was presented by Crognale [12] near zero dispersion wavelength. Recalling the small signal approach reported in [11], a simple and exhaustive treatment was developed to study the small-signal and noise transmission characteristics in the frequency domain of a high performance laser diode together with a linear dispersive fiber. The impact of second-order dispersion term on the modulation and noise properties of an ultrafast laser diode was obtained to study the total frequency response and the relative intensity noise (RIN) at output of the linear single-mode fiber but the impact of other higher-order dispersion terms was neglected.

In this paper, we have extended the work reported in [11,12] by presenting an improved analysis for analyzing the influence of the higher-order dispersion on dispersive optical communication system. A modified conversion matrix has been reported that gives the transfer function of intensity and phase from the fiber input to the fiber output for any laser source including the influence of any higher-order dispersion terms. Moreover, this theory is applicable to evaluate the impact of higher-order dispersion on the small-signal frequency response and RIN of an ultrafast laser diode similarly as mentioned in [12]. In addition, a generalized conversion matrix is presented to analyze the small-signal and noise transmission characteristics of ultrafast laser diode through a dispersive medium by incorporating the influence of any order of dispersion term.

First, in Section 2 we present the modified analysis up to fifth-order dispersion term for a phase and intensity of modulated signal propagating through a dispersive medium, yielding the desired generalized conversion matrix inclusive influence of higher-order dispersion terms. Section 3 discusses the small signal analysis leading to generalized conversion matrix. Section 4 discusses the application of conversion matrix for small-signal intensity and frequency modulation characteristics of laser diode and RIN at output of a dispersive fiber including the impact of higher-order dispersion. In Section 5, the various responses are plotted and results are obtained, and finally in Section 6, the conclusions are drawn.

## 2. Analysis for intensity and phase of an optical field at the fiber output

In this section we will present a transfer function using similar approach reported in [11,12] for a linear dispersive single-mode fiber, including the effects of the higher-order dispersion terms, i.e., second-, third-, fourth-, and fifth-order dispersion terms. In fact, the chromatic dispersion is the most significant limiting factor to degrade the performance of ultrafast long-distance broadband optical communication systems. Therefore, it is important not only to analyze the impact of second-order dispersion, but also its slope (third-order dispersion, 3OD), curvature (fourth-order dispersion, 4OD) [13], and quadrature (fifth-order dispersion, 5OD) for ultra fast long-distance broadband optical communication systems.

Let the electric field at the input of fiber from a single mode laser diode [14] be

$$E(t) = E_{\mathrm{in}}(t)e^{j\omega_0 t} \tag{1}$$

with the slowly varying complex amplitude $E_{\text{in}}(t)$ and the mean optical frequency $\omega_0$ which is given by [11]

$$E_{\text{in}}(t) = \sqrt{S_{\text{in}}(t)}e^{j\phi_{\text{in}}(t)}, \tag{2}$$

where $S_{\text{in}}(t)$ and $\phi_{\text{in}}(t)$ are, respectively, the input photon intensity and input phase. As in [15], the propagation of the signal through an optical fiber can be described by propagation term $e^{-j\beta L}$ with length $L$ of the transmission fiber and the phase constant $\beta$ (losses are neglected) by the relation

$$E_{\text{out}}(\omega) = E_{\text{in}}(\omega)e^{-j\beta L}, \tag{3}$$

where the propagation constant $\beta$ in terms of Taylor series can be expanded around $\omega = \omega_0$ as mentioned in [15–18] as

$$\beta = \beta_0 + (\omega-\omega_0)\tau + \frac{1}{2}(\omega-\omega_0)^2\frac{d\tau}{d\omega} + \frac{1}{6}(\omega-\omega_0)^3\frac{d^2\tau}{d\omega^2} + \frac{1}{24}(\omega-\omega_0)^4\frac{d^3\tau}{d\omega^3} + \frac{1}{120}(\omega-\omega_0)^5\frac{d^4\tau}{d\omega^4} + \cdots. \tag{4}$$

We define the following dispersion parameters:

$$F_2 = -\frac{L}{2}\frac{d\tau}{d\omega} = \frac{L}{2}\frac{\lambda^2}{2\pi c}\frac{\partial\tau}{\partial\lambda} \tag{5}$$

for second-order dispersion,

$$F_3 = \frac{L}{6}\frac{d^2\tau}{d\omega^2} = \frac{L}{6}\frac{\lambda^2}{(2\pi c)^2}\left[\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 2\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{6}$$

for third-order dispersion,

$$F_4 = -\frac{L}{24}\frac{d^3\tau}{d\omega^3} = \frac{L}{24}\frac{\lambda^3}{(2\pi c)^3}\left[\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 6\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 6\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{7}$$

for fourth-order dispersion, and

$$F_5 = \frac{L}{120}\frac{d^4\tau}{d\omega^4} = \frac{L}{120}\frac{\lambda^4}{(2\pi c)^4}\left[\lambda^4\frac{\partial^4\tau}{\partial\lambda^4} + 12\lambda^3\frac{\partial^3\tau}{\partial\lambda^3} + 36\lambda^2\frac{\partial^2\tau}{\partial\lambda^2} + 24\lambda\frac{\partial\tau}{\partial\lambda}\right] \tag{8}$$

for fifth-order dispersion.

As reported in [13–16], we neglect the absolute phase and group delay ($\phi_0 = \beta_0 L$ and $d\beta/d\omega = \tau$) because both terms produce only phase delay of the carrier signal and have no influence on the distortion of the signal. Therefore, in frequency domain

$$E_{\text{out}}(j\omega) = E_{\text{in}}(j\omega)e^{jF_2(\omega-\omega_0)^2 - jF_3(\omega-\omega_0)^3 + jF_4(\omega-\omega_0)^4 - jF_5(\omega-\omega_0)^5 + \cdots}. \tag{9}$$

In time domain, since $j\omega = \partial/\partial t$, $(j\omega)^2 = -\omega^2 = \partial^2/\partial t^2$, and $(j\omega)^3 = -j\omega^3 = \partial^3/\partial t^3$, etc.,

$$E_{\text{out}}(t) = e^{j\omega_0 t}e^{\left(-jF_2\frac{\partial^2}{\partial t^2} + F_3\frac{\partial^3}{\partial t^3} - jF_4\frac{\partial^4}{\partial t^4} + F_5\frac{\partial^5}{\partial t^5} - \cdots\right)}\sqrt{S_{\text{in}}(t)}e^{j\phi_{\text{in}(t)}}. \tag{10}$$

Let

$$E_{out}(t) = E_{in}(t) + \Delta E(t), \tag{11}$$

where

$$\left|\Delta E(t)\right| \ll \left|E_{in}(t)\right|. \tag{12}$$

From Eqs. (10) and (11)

$$\Delta E(t) = \left(e^{(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots)} - 1\right)\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}. \tag{13}$$

From (11) and with the help of approximation (12), we have

$$S_{out}(t) = \left|E_{in}(t) + \Delta E(t)\right|^2 \approx \left|E_{in}(t)\right|^2 + 2\Re\left[E_{in}^*(t)\Delta E(t)\right]. \tag{14}$$

Substituting Eqs. (2) and (13) in Eq. (14)

$$S_{out} = S_{in}(t) + 2\Re\Big[\sqrt{S_{in}(t)}e^{-j\phi_{in}(t)}\left(e^{(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots)} - 1\right) \times \sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\Big], \tag{15}$$

$$\phi_{out}(t) = \phi_{in}(t) + \Im\left[\frac{\Delta E(t)}{E_{in}(t)}\right], \tag{16}$$

$$\phi_{out}(t) = \phi_{in}(t) + \Im\left[\frac{\left(e^{(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots)} - 1\right)\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}}{\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}}\right]. \tag{17}$$

Equations (15) and (17) derived above are the general equations describing the intensity and phase of an optical field after propagating through a dispersive optical fiber. These equations are valid for arbitrary input intensity and phase as long as dispersion induced field amplitude $\Delta E(t)$ is assumed to be small compared to the input field $E_{in}(t)$. Further, the expressions (15) and (17) can be simplified using small-signal analysis.

## 3. Small-signal analysis

The small-signal analysis implies that frequency modulation or noise $\dot{\phi} = d\phi/dt$ and the average intensity $\langle S\rangle$ is larger then the noise or modulation term $\Delta S_{in}(t)$:

$$S_{in}(t) = \langle S\rangle + \Delta S_{in}(t) \tag{18}$$

with satisfying following relation $\langle S\rangle \gg \Delta S_{in}(t)$.

As reported in [11,12] in the small-signal approach, the field amplitude $\sqrt{S_{in}(t)}$ can be linearized as

$$\sqrt{S_{in}(t)} \approx \sqrt{\langle S\rangle}\left(1 + \frac{\Delta S_{in}(t)}{2\langle S\rangle}\right). \tag{19}$$

After neglecting the product of small-signal [11,12,20], we can introduce following approximations:

$$\frac{\partial^n}{\partial t^n}\left(\sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\right) \approx \sqrt{S_{in}(t)}e^{j\phi_{in}(t)}\frac{\partial^n}{\partial t^n}\sqrt{\langle S\rangle}\left(j\phi_{in}(t) + \frac{\Delta S_{in}(t)}{2\langle S\rangle}\right). \tag{20}$$

Inserting (20) into (15) and (17), we obtain

$$S_{\text{out}}(t) = S_{\text{in}}(t) + 2\Re\left[\langle S\rangle\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots\right)}\right)\times\left(\frac{\Delta S_{\text{in}}(t)}{2\langle S\rangle}+j\phi_{\text{in}}(t)\right)\right] \tag{21}$$

with $S_{\text{in}}(t) \approx \langle S\rangle$ because $\langle S\rangle \gg \Delta S_{\text{in}}(t)$ and similarly

$$\phi_{\text{out}}(t) = \Im\left[\left(e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots\right)}\right)\left(\frac{\Delta S_{\text{in}}(t)}{2\langle S\rangle}\right)+j\phi_{\text{in}}(t)\right)\right]. \tag{22}$$

The exponential operator can be written as [12]

$$e^{\left(-jF_2\frac{\partial^2}{\partial t^2}+F_3\frac{\partial^3}{\partial t^3}-jF_4\frac{\partial^4}{\partial t^4}+F_5\frac{\partial^5}{\partial t^5}-\cdots\right)} \approx e^{-j\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)}e^{\left(F_3\frac{\partial^3}{\partial t^3}+F_5\frac{\partial^5}{\partial t^5}\right)} \approx e^{-j\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)}\left(1+F_3\frac{\partial^3}{\partial t^3}+F_5\frac{\partial^5}{\partial t^5}\right). \tag{23}$$

We use approximation assuming $F_3$ and $F_5$ to be very small and expressing $e^x = 1 + x$. (The expansion has been carried out only up to first term, the higher-order terms being ignored.) Also, since $e^{-jx} = \cos(x) - j\sin(x)$, we get

$$= \left[\cos\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right) - j\sin\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)\right]\left(1+F_3\frac{\partial^3}{\partial t^3}+F_5\frac{\partial^5}{\partial t^5}\right). \tag{24}$$

In this way the relations for intensity and phase derived at the fiber output are

$$\Delta S_{\text{out}}(t) = \cos\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)\left[\Delta S_{\text{in}}(t) + F_3\Delta\dddot{S}_{\text{in}}(t) + F_5\Delta\overset{\cdot\cdot\cdot\cdot\cdot}{S}_{\text{in}}(t)\right] + 2\langle S\rangle\sin\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)\left[\phi_{\text{in}}(t) + F_3\dddot{\phi}_{\text{in}}(t) + F_5\overset{\cdot\cdot\cdot\cdot\cdot}{\phi}_{\text{in}}(t)\right], \tag{25}$$

$$\phi_{\text{out}}(t) = \cos\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)\left[\phi_{\text{in}}(t) + F_3\dddot{\phi}_{\text{in}}(t) + F_5\overset{\cdot\cdot\cdot\cdot\cdot}{\phi}_{\text{in}}(t)\right] - 2\sin\left(F_2\frac{\partial^2}{\partial t^2}+F_4\frac{\partial^4}{\partial t^4}\right)\left[\frac{\Delta S_{\text{in}}(t)}{2\langle S\rangle} + F_3\frac{\Delta\dddot{S}_{\text{in}}(t)}{2\langle S\rangle} + F_5\frac{\Delta\overset{\cdot\cdot\cdot\cdot\cdot}{S}_{\text{in}}(t)}{2\langle S\rangle}\right], \tag{26}$$

with the output modulation or noise given by

$$\Delta S_{\text{out}}(t) = S_{\text{out}}(t) - \langle S\rangle \tag{27}$$

and dots denote the time derivatives. Recalling the relation between frequency and phase [11,12] and taking practical systems where we consider frequency modulation or noise $\dot{\phi} = d\phi/dt$ rather than phase modulation, results may be expressed with modified conversion matrix that describes the relation between intensity and frequency, including the second-, third-, fourth-, and fifth-order dispersion terms as shown in expressions (25) and (26) and is the extension of the expression reported in [11,12]:

$$\begin{pmatrix} \Delta S_{\text{out}} \\ \dot{\phi}_{\text{out}} \end{pmatrix} = \begin{bmatrix} \cos(F_2\omega^2 + F_4\omega^4) & 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4) \\ \quad \cdot (1 - jF_3\omega^3 - jF_5\omega^5) & \quad \cdot \left(\frac{j}{\omega} + F_3\omega^2 + F_5\omega^5\right) \\ \frac{\sin(F_2\omega^2+F_4\omega^4)}{2\langle S\rangle} & \cos(F_2\omega^2 + F_4\omega^4) \\ \quad \cdot (j\omega + F_3\omega^4 + F_5\omega^6) & \quad \cdot (1 - j\omega^3 F_3 - j\omega^5 F_5) \end{bmatrix} \begin{pmatrix} \Delta S_{\text{in}}(\omega) \\ \dot{\phi}_{\text{in}}(\omega) \end{pmatrix}. \tag{28}$$

The modified conversion matrix (28) derived above may be generalized for other higher-order dispersion terms. The generalized form of the conversion matrix may be written as under and is valid to derive conversion matrices reported in [11,12] and all previous results. Also the new results can be derived by including or excluding the respective higher-order dispersion terms:

$$\begin{pmatrix} \Delta S_{\text{out}}(\omega) \\ \dot{\phi}_{\text{out}}(\omega) \end{pmatrix} = \begin{bmatrix} \cos(F_2\omega^2 + F_4\omega^4 + \cdots + F_{2n}\omega^{2n}) & 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4 + \cdots + F_{2n}\omega^{2n}) \\ \quad \cdot (1 - jF_3\omega^3 - jF_5\omega^5 - \cdots - jF_{2n+1}\omega^{2n+1}) & \quad \cdot \left(\frac{j}{\omega} + F_3\omega^2 + \cdots + F_{2n+1}\omega^{2n}\right) \\ \frac{\sin(F_2\omega^2+F_4\omega^4+\cdots+F_{2n}\omega^{2n})}{2\langle S\rangle} & \cos(F_2\omega^2 + F_4\omega^4 + \cdots + F_{2n}\omega^{2n}) \\ \quad \cdot (j\omega + F_3\omega^4 + F_5\omega^6 + \cdots + F_{2n+1}\omega^{2n+2}) & \quad \cdot (1 - j\omega^3 F_3 - \cdots - j\omega^{2n+1} F_{2n+1}) \end{bmatrix} \times \begin{pmatrix} \Delta S_{\text{in}}(\omega) \\ \dot{\phi}_{\text{in}}(\omega) \end{pmatrix}. \tag{29}$$

If we assume that only intensity modulation is present and there is no phase modulation, we can obtain transfer function $\cos(F_2\omega^2 + F_4\omega^4)(1 - jF_3\omega^3 - jF_5\omega^5)$ in comparison with $\cos(F_2\omega^2)$ [11] obtained exclusive third-, fourth-, and fifth-order dispersion terms.

On the other hand, if we assume that only phase modulation is present, the intensity modulation $S_{\text{out}}(\omega)$ at output of a dispersive fiber due to FM–AM conversion can be expressed as

$$\Delta S_{\text{out}}(\omega) = 2\langle S\rangle \sin(F_2\omega^2 + F_4\omega^4)\left(\frac{j}{\omega} + F_3\omega^2 + F_5\omega^4\right). \tag{30}$$

Using above analysis it is easy to calculate the transfer function in the presence of either only intensity modulation or phase modulation through a linear dispersive fiber including the influence of higher-order dispersion terms. In addition, Eq. (29) does not need the calculation of Bessel function.

## 4. Small-signal intensity and frequency modulation characteristics and RIN at output of a dispersive single mode fiber

If we consider a modulation of the injection current around the mean value $\langle I\rangle$, the small-signal response of the intrinsic laser may be obtained with small-signal modulation current $\Delta I$, so that $|\Delta I| \ll \langle I\rangle$, yielding in frequency domain [12]

$$\Delta S_{\text{in}}(\omega) = \left(\frac{\tau_{\text{ph}}}{e}\right) H(\omega)\Delta I(\omega), \tag{31}$$

where $\tau_{ph}$ is the photon lifetime, $e$ is the elementary charge. The effective life time $\tau_e'$ is defined as

$$\frac{1}{\tau_e'} = \frac{1}{\tau_e} + \omega_r^2 \tau_{ph} \tag{32}$$

with $\tau_e$ as the carrier lifetime and $\omega_r$ as relaxation resonance frequency. Also, the small-signal modulation transfer function is given by [12]

$$H(\omega) = \frac{\omega_r^2}{(j\omega)^2 + j\omega\Gamma + \omega_r^2}. \tag{33}$$

Here, the $\Gamma$ is the damping rate. The relation between frequency modulation and intensity modulation (chirp) may be described by [14,21]

$$\dot{\phi}_{in}(\omega) = \frac{\alpha}{2}(j\omega + \omega_g)\frac{\Delta S_{in}(\omega)}{\langle S \rangle}, \tag{34}$$

where $\alpha$ is the linewidth enhancement factor, and $\omega_g$ is the device-specific characteristic frequency. If the chirp characteristics are mainly due to nonlinear gain, we have $\omega_g \approx \Gamma$. The intensity modulation at the fiber output can be obtained by inserting Eqs. (31) and (34) into (28):

$$\frac{\Delta S_{out}}{\Delta I(\omega)} = \left(\frac{\tau_{ph}}{e}\right)(1 - j\omega^3 F_3 - j\omega^5 F_5) \times \left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] H(\omega). \tag{35}$$

Expression (35) gives intensity modulation at output of a dispersive fiber including the effects of second-, third-, fourth-, and fifth-order dispersion terms. Recalling the results reported in [11,12], all these results of small signal response can be derived from this expression. With third- and fifth-order dispersion terms equal to zero, we have

$$\left[\frac{\Delta S_{out}}{\Delta I(\omega)}\right]_{F_3=0,\, F_5=0} = \left(\frac{\tau_{ph}}{e}\right)\left[\cos(F_2\omega^2 + F_4\omega^4) + j\alpha \sin(F_2\omega^2 + F_4\omega^4)\frac{j\omega + \omega_g}{\omega}\right] H(\omega), \tag{36}$$

$$\left[\frac{\Delta S_{out}}{\Delta I(\omega)}\right] = (1 - j\omega^3 F_3 - j\omega^5 F_5)\left[\frac{\Delta S_{out}}{\Delta I(\omega)}\right]_{F_3=0,\, F_5=0}. \tag{37}$$

Equation (37) shows that at high modulation frequencies the third- and fifth-order dispersion term introduces an enhancement factor

$$\left|\frac{\Delta S_{out}}{\Delta I(\omega)}\right| = \left(1 + \omega^6 F_3^2 + \omega^{10} F_5^2 + 2\omega^8 F_3 F_5\right)^{1/2} \left|\frac{\Delta S_{out}}{\Delta I(\omega)}\right|_{F_3=0,\, F_5=0}. \tag{38}$$

In addition, this small-signal analysis may be applied to evaluate the RIN of an ultrafast laser diode similarly as reported in [11,12] in order to see the influence of higher-order dispersion terms all together and independently. The semiconductor laser noises with spontaneous emission noises are considered here.

Assuming the laser noise induced predominantly by the spontaneous emission noise, the Langevin noise sources for intensity and phase are given by [11,14]

$$\langle |F_S(\omega)|^2 \rangle = 2R\langle S\rangle, \tag{39}$$

$$\langle |F_\phi(\omega)|^2 \rangle = \frac{R}{2\langle S\rangle}, \tag{40}$$

where $R$ and $\langle S\rangle$ are spontaneous emission rate and average photon density, respectively, of the semiconductor laser diode with

$$\langle \Delta F_S(\omega) \Delta F_\phi^*(\omega) \rangle = 0. \tag{41}$$

If the laser noise is assumed to be solely by spontaneous emission noise [11], we have

$$\Delta S(\omega) = \left( \frac{\frac{1}{\tau_e'} + j\omega}{\omega_r^2} \right) H(\omega) \Delta F_S(\omega), \tag{42}$$

$$\dot{\phi}(\omega) = \frac{-\alpha\omega_R^2}{2\left(j\omega + \frac{1}{\tau_e'}\right)} \frac{\Delta S(\omega)}{\langle S\rangle} + \Delta F_\phi(\omega). \tag{43}$$

It is useful to introduce a “relative intensity noise” (RIN) relating the intensity fluctuations $\Delta S(\omega)$, referred to a noise bandwidth $\Delta f$, to mean intensity $\langle S\rangle$. By definition [12],

$$\frac{\mathrm{RIN}}{\Delta f} = \frac{2\langle |\Delta S(\omega)|^2 \rangle}{\langle S\rangle^2}. \tag{44}$$

From (42), the intrinsic RIN of semiconductor laser may be written as

$$\frac{\mathrm{RIN}}{\Delta f} = \frac{4R}{\langle S\rangle} \left( \frac{\frac{1}{\tau_e'^2} + \omega^2}{\omega_r^4} \right) |H(\omega)|^2. \tag{45}$$

It is well known that RIN is significantly enhanced after passing through dispersive fiber because of different delays of the spectral components within the spectral width of the laser and due to higher-order dispersion terms.

The FM–AM noise conversion with the intensity and phase noise of the laser source has been investigated in [19–21]. Recalling the results obtained in [11,12], and the modified conversion matrix (29), one finally obtains the RIN at the fiber output due to higher-order dispersion terms by inserting Eqs. (42), (43) in (28) and recalling (45):

$$\begin{aligned} \frac{\mathrm{RIN}}{\Delta f} = {} & \frac{4R}{\langle S\rangle} \left(1 + F_3^2\omega^6 + F_5^2\omega^{10} + 2\omega^8 F_3 F_5\right) \\ & \times \left\{ \frac{1+\omega^2\tau_e'^2}{\omega_r^4 \tau_e'^2} |H(\omega)|^2 \cos^2(\omega^2 F_2 + \omega^4 F_4) \right. \\ & \quad + \left(\alpha^2 |H(\omega)|^2 + 1\right) \frac{\sin^2(\omega^2 F_2 + \omega^4 F_4)}{\omega^2} \\ & \quad \left. - \frac{\alpha \sin^2(2\omega^2 F_2 + 2\omega^4 F_4)}{\omega_r^2} |H(\omega)|^2 \right\}. \end{aligned} \tag{46}$$

Results reported in [11] and [12] for intensity modulation and RIN at the fiber output may also be derived using modified conversion matrix to analyze the impact of higher-order dispersion terms. The modified conversion matrix reported in this paper permits to derive, for any arbitrary frequency and intensity modulation (or noise) at the fiber input, the corresponding frequency and intensity modulation (or noise) at the fiber output, taking also into account the correlation between the phase and intensity modulation and considering the higher-order dispersion effects.

## 5. Results and discussion

Referring to ITU:T Recommendations G.653 [22] up to fourth-order dispersion term, we assume that for DS fiber near 1550 nm

$$\frac{d\tau}{d\lambda} = (\lambda - \lambda_0)S_0, \tag{55}$$

in which $S_0 = d^2\tau/d\lambda^2$ is zero dispersion slope and $\lambda_0$ is zero dispersion wavelength. Imposing $\lambda_0 = 1550$ nm and $S_0 = 0.085$ ps/nm$^2$/km, we have $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.000618$ ps$^4$/km; other semiconductor laser parameters are relaxation resonance frequency $f_R = 20.25$ GHz, damping rate $\gamma = 63.29$ GHz, spontaneous emission rate $R = 2.54 \times 10^{12}$ s$^{-1}$, average photon density $\langle S\rangle = 4.5 \times 10^5$, photon life time $\tau_{\text{ph}} = 118 \times 10^{-12}$ s, $\tau_e = 0.17 \times 10^{-9}$ s, linewidth enhancement factor $\alpha = 5$, and maximum intrinsic modulation frequency $f_{\max} = 63.47$ GHz. We plot the frequency response by taking Eq. (35) for various different combination cases of second-, third-, and fourth-order dispersion terms as shown in Figs. 1–7. We discuss here two cases, $L = 0$ km which indicates the frequency response at fiber input and $L = 10{,}000$ km which indicates the response at fiber output after propagation of the signal. The plot with second-order dispersion term is shown in Fig. 1. As is clear from the figure, the frequency response with F2 only at $L = 10{,}000$ km vary from response at $L = 0$ at high modulation frequencies. The response with $L = 0$ km is almost linear over wide range of modulating frequencies but with $L = 10{,}000$ km, it no longer remains linear. The responses with third- and fourth-order dispersion terms only are shown in Figs. 2 and 3. The deviations from second-order dispersion are clearly noticed reflecting the impact of each term independently. Thus, if the impact of second-order dispersion is zero, then the impact of higher-order dispersion terms is very important. Figure 4 shows the response with combined effect of second- and third-order dispersion terms. The variations from earlier case with F2 only (Fig. 1) is observed for this case. Similarly, the plot for second- and fourth-order dispersion terms together is shown in Fig. 5 indicating minute variations. With second-, third-, and fourth-order dispersion terms together, again there are deviations as shown in Fig. 6. Figure 7 indicates the important comparison of results including F2. It is clear from the figure that third-order dispersion term has significant impact on frequency response at large propagation distances and at long distance links. The results agree with the results reported in [12]. The additional impact of F4 is also observed for this analysis in the figure with minute deviation as compared to combined effect of F2 and F3. It is also seen that the drop in frequency response for F2 only and with F2 and F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at slightly low frequency. This reflects that each term has its own impact on the small-signal frequency response at high modulating frequencies and large propagation distances.

For the same set of parameters, we now plot the relative intensity noise response $\text{RIN}/\Delta f$ with modulation frequencies according to Eq. (46) for various combination cases of second-, third-, and fourth-order dispersion terms as shown in Figs. 8–14. We again discuss two cases $L = 0$ km and $L = 10{,}000$ km. The plot for RIN with second-order dispersion term is shown in Fig. 8. It is seen that the response varies for input ($L = 0$ km) and output ($L = 10{,}000$ km) of single mode fiber at high modulating frequencies and large propagating distances. It is also noticed that RIN response at $L = 10{,}000$ km drops at

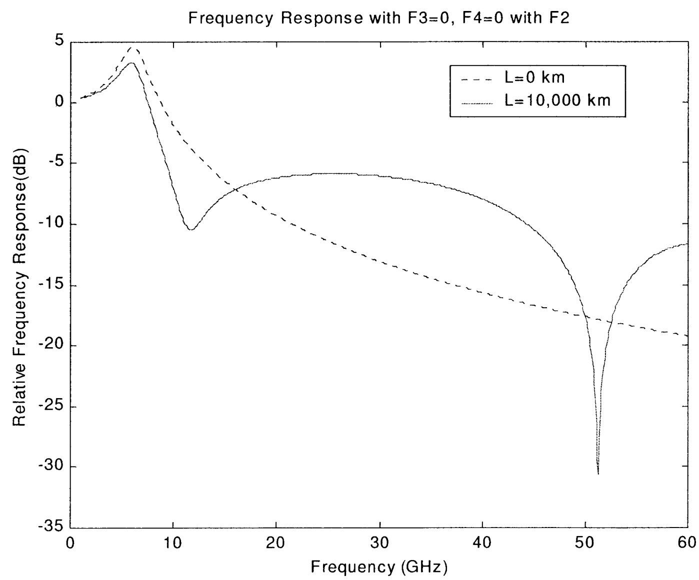


Fig. 1. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 only.

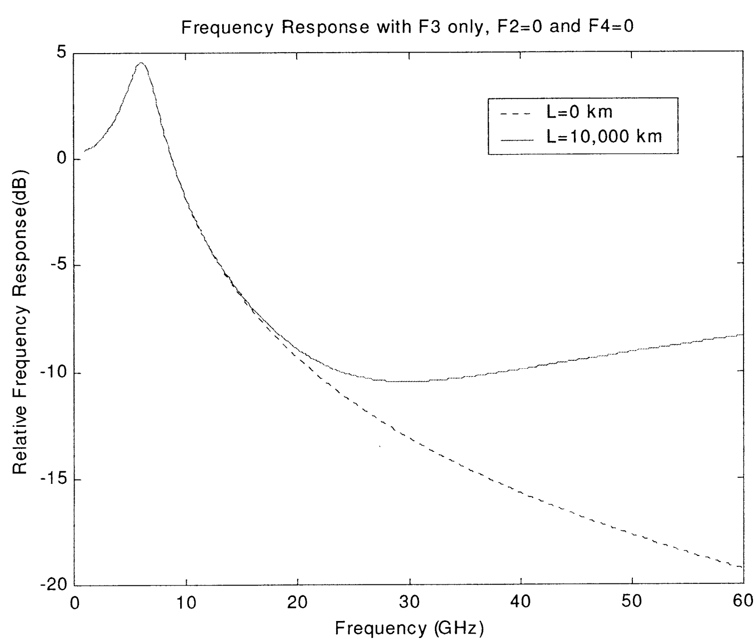


Fig. 2. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F3 only.

approximately 57 GHz frequency and further increases with more increase in frequency. The plots with third- and fourth-order dispersion terms independently are shown in Figs. 9 and 10. The variations are clearly visible from the figure indicating the independent impacts. With second- and third-order dispersion terms together, the deviation is noticed

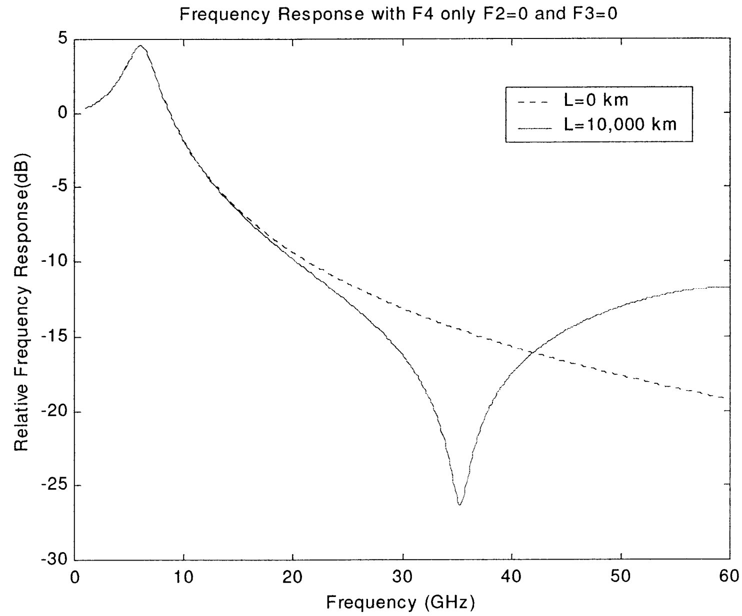


Fig. 3. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F4 only.

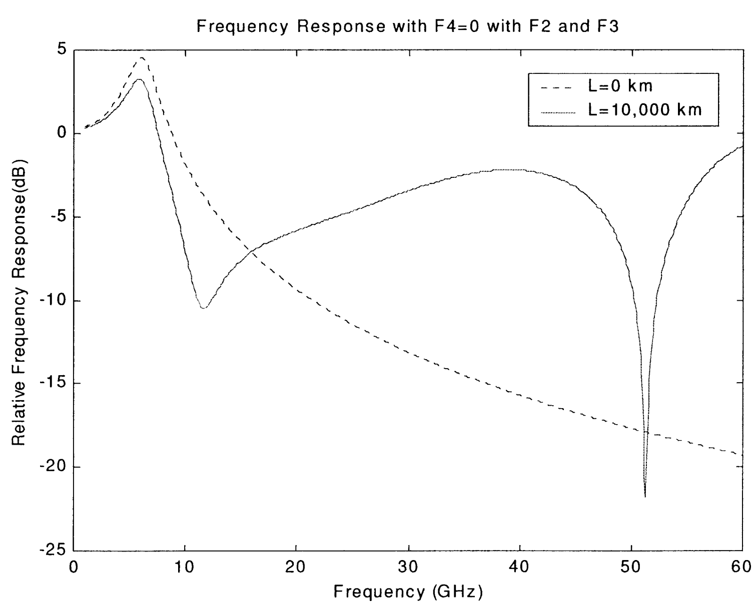


Fig. 4. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 and F3.

from earlier case (Fig. 8) as shown in Fig. 11. The plot for second- and fourth-order dispersion terms together is shown in Fig. 12. With second-, third-, and fourth-order dispersion terms together, the additional impact of F4 is also observed as shown in Fig. 13. Figure 14 indicates the combined impact of these three cases. It is clear from

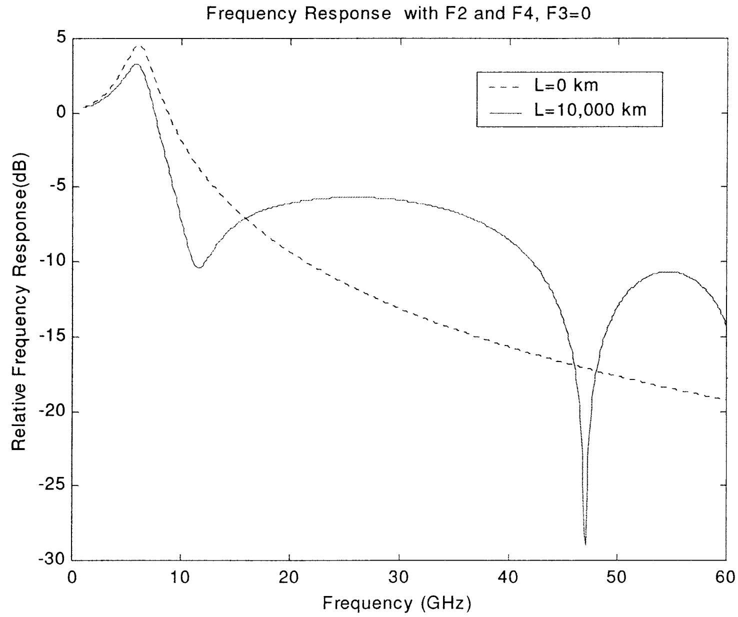


Fig. 5. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 and F4.

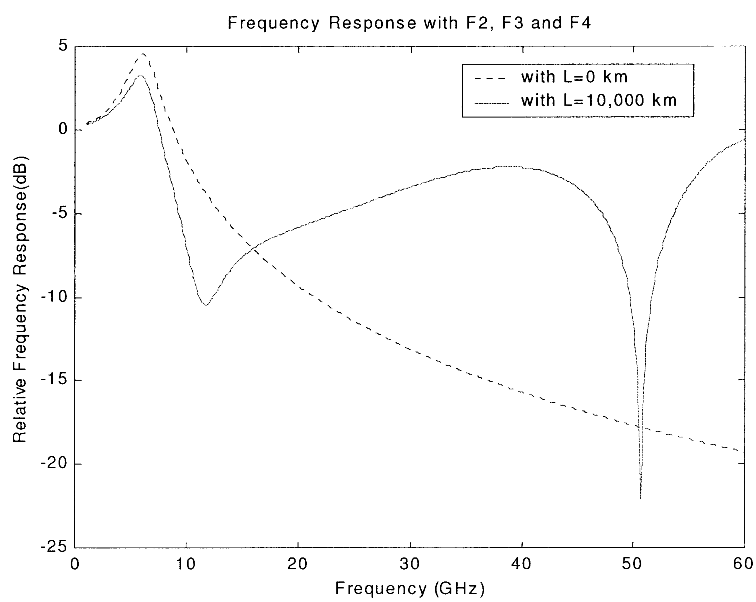


Fig. 6. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2, F3, and F4.

the figure that third-order dispersion term has significant impact on frequency response at large propagation distances at long distance links. The results again agree with the results reported in [12]. The additional impact of F4 is also seen in the figure with minute deviations. It is again seen that the drop in frequency response for F2 only and with F2 and

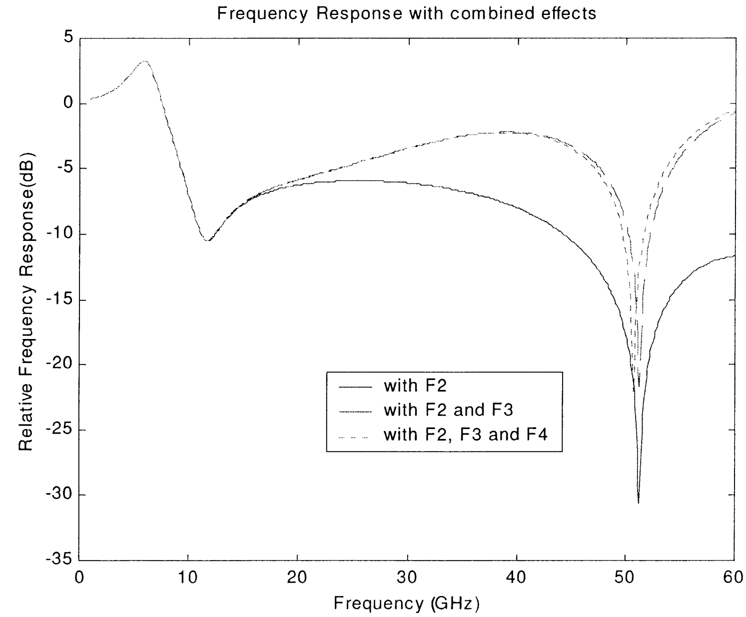


Fig. 7. Small-signal frequency response according to Eq. (35) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. Combined effects.

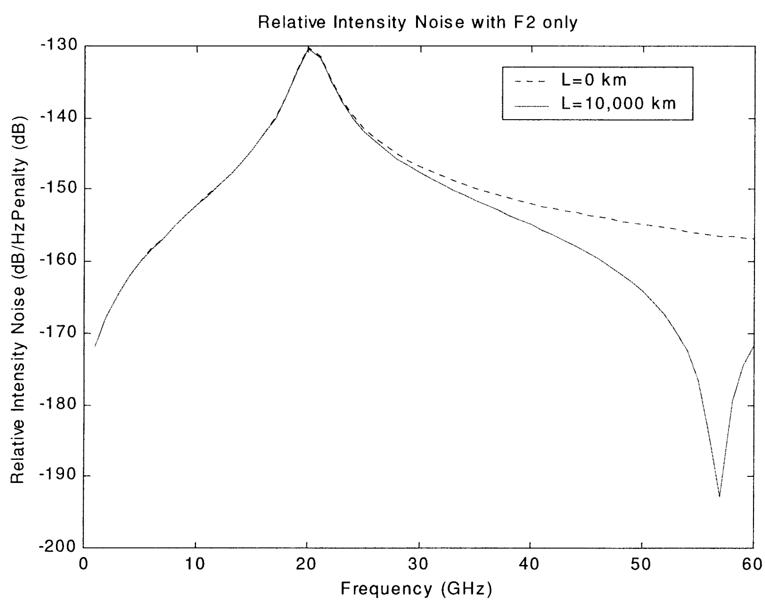


Fig. 8. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 only.

F3 together is at the same frequency whereas this drop for F2, F3, and F4 together is at slightly high frequency. This again reflects that each term has its own impact on the small-signal frequency response at high modulating frequencies and large propagation distances for RIN also.

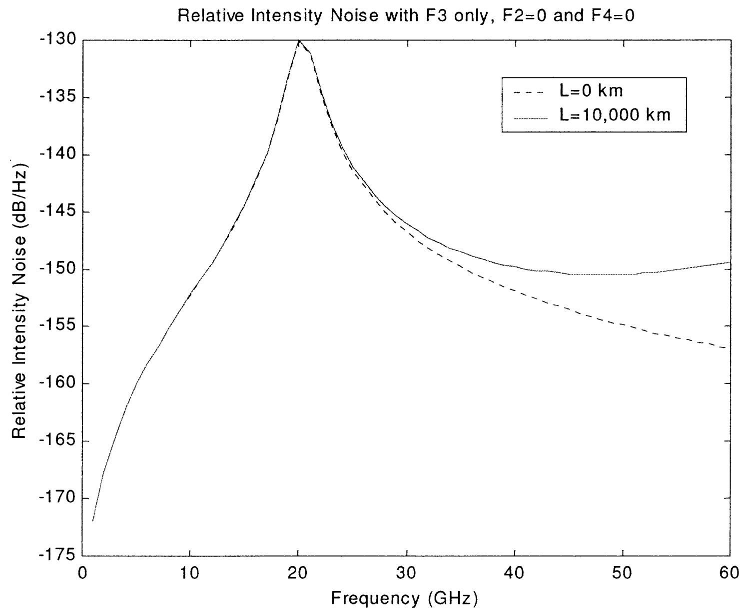


Fig. 9. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F3 only.

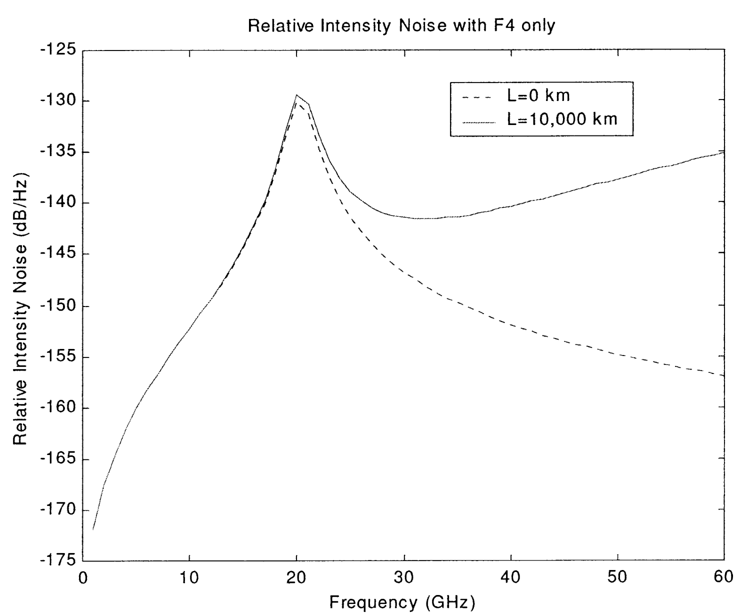


Fig. 10. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F4 only.

## 6. Conclusions

In this paper we have developed the theory to investigate the influence of higher-order dispersion terms on optical communication systems using small-signal analysis near zero

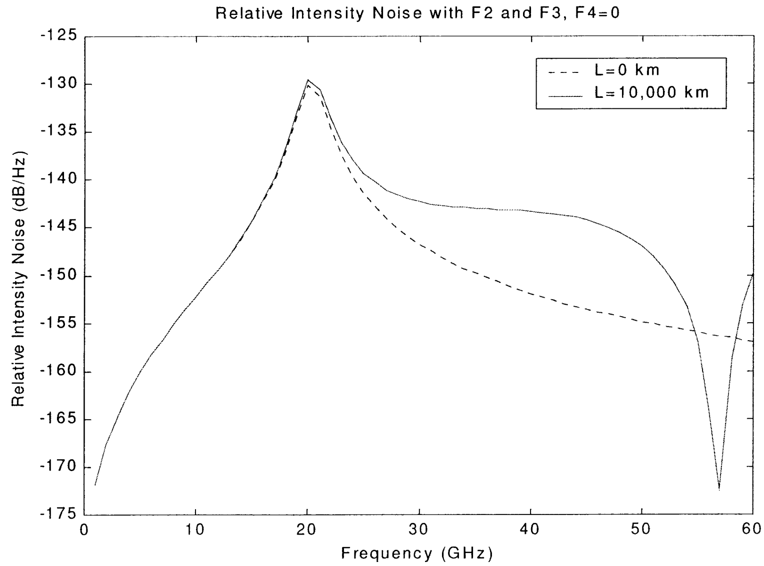


Fig. 11. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 and F3.

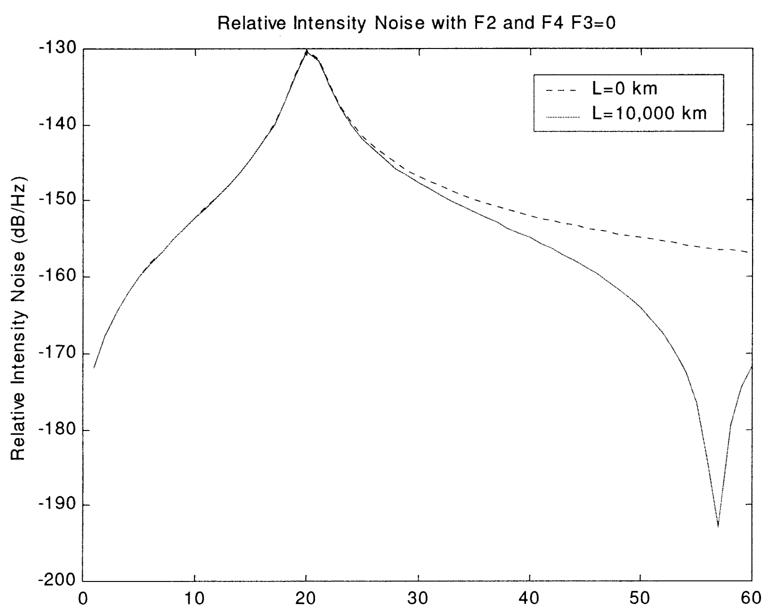


Fig. 12. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2 and F4.

dispersion wavelength. The theory developed in this paper may be considered as extension of the results reported in [11,12], where the impact of higher-order dispersion terms was neglected. The transfer function of intensity and phase from the fiber input to fiber output for every source emission wavelength inclusive the impact of any high-order dispersion terms may be obtained using modified conversion matrix. Results have been presented

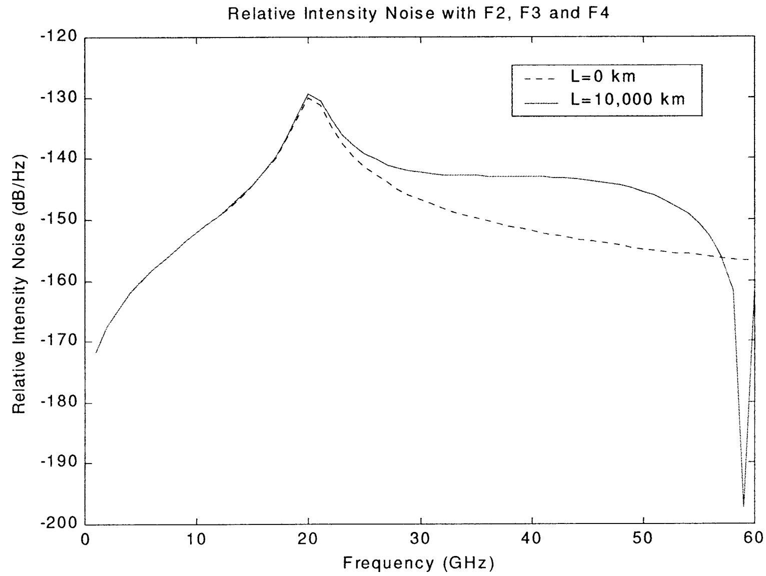


Fig. 13. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. F2, F3, and F4.

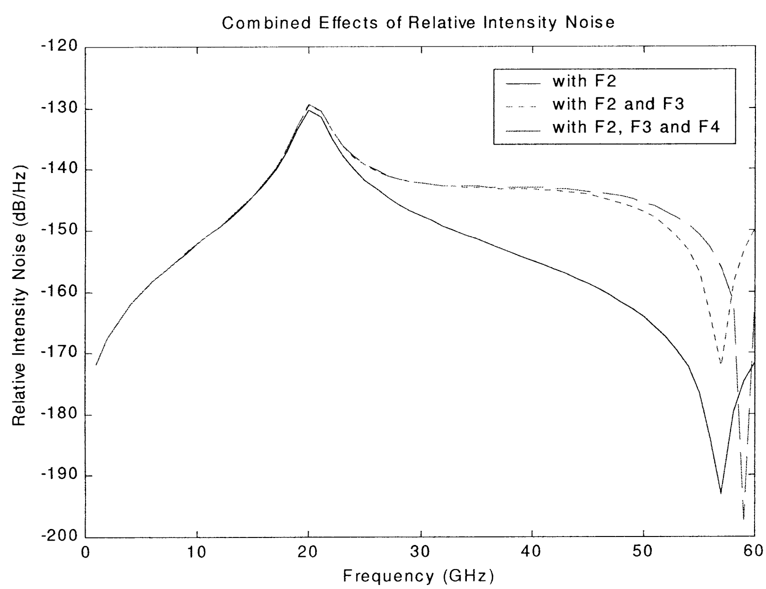


Fig. 14. Small-signal relative intensity noise (RIN) response according to Eq. (46) for $L = 0$ km and $L = 10{,}000$ km with $S_0 = 0.085$ ps/nm$^2$/km, $d\tau/d\lambda = 5 \times 10^{-3}$ ps/nm/km, $d^2\tau/d\omega^2 = 0.138$ ps$^3$/km, and $d^3\tau/d\omega^3 = 0.1637$ ps$^4$/km. Combined effects.

to obtain the small-signal frequency response of phase and intensity modulation of a laser diode together in a dispersive fiber. In addition, a generalized conversion matrix is presented to present all previous and new results up to any order of dispersion term. Further, it has been investigated that the higher-order dispersion terms introduce an enhancement factor of transfer function and RIN characteristics at very high modulation frequencies. The relative frequency response and relative intensity noise response is plotted for second-, third-, and fourth-order dispersion terms and their combinations. It is seen that the higher-order dispersion terms have significant impact on frequency and RIN responses at large modulating frequencies and large propagation distances. Further, it is also seen that as the order of dispersion term increases, its impact becomes lesser and lesser but if the second-order dispersion term is negligible, the higher-order terms are of significant relevance at long links or high frequencies.

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