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Higher-Order Dispersion Compensation by Differential Time Delay

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A novel approach to combating the pulse broadening effects of group-velocity dispersion in a fiber-optic communication link due to higher-order dispersion terms has been presented. It has been shown that it is possible for a short pulse to propagate without significant broadening over lengths many times longer than the usual dispersion length of fiber. Root mean square phase deviation, dimension-free chirp parameter, and figure of merit have been evaluated for ideal and realistic optical communication systems using higher-order dispersion terms together. It has been shown that dispersionless propagation length can be enhanced to 5 and 6.5 times if the compensation is performed by using second- and third-order and third- and fourth-order dispersion terms together, respectively. © 1998 Academic Press

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1. INTRODUCTION

Recently, there has been great interest in using single-mode fibers for high-bit-rate transmission in low-loss transmission windows but dispersion plays an important role in degrading the overall system performance of an optical communication system. At high bit rate, the dispersion-induced broadening of short pulses propagating in the fiber causes crosstalk between the adjacent time slots, leading to errors when the communication distance increases beyond the dispersion length of the fiber. Higher-order dispersion terms are the forces destructive to pulse propagation in ultra-high-bit-rate optical transmission system. Therefore, in order to realize the high data rates over long distances down the SM fiber, techniques must be found to overcome the pulse spreading due to dispersion. Various methods for dispersion compensation have been reported in [1−15].

A technique for the synthesis of an optical signal predistorted to compensate for fiber dispersion was discussed in [2], but a prechirp technique is the most common way to deal with fiber dispersion. The prechirp technique is based on a predistortion method [1, 2] and it is possible to expand the allowable transmission length over two times [3], but the dispersion cannot be compensated thoroughly. A modified prechirp technique, which utilizes time division superimposing prechirp bit streams, has been mentioned in [12] to achieve greater dispersion compensation capability. The allowable transmission dispersion is nearly four times greater than that for the systems without dispersion compensation technique.

An attempt has been made to reduce fiber chromatic dispersion using an optical external modulator with adjustable and accurately controllable chirp [4].

There is another technique which is based on hetrodyne mixing and dispersion compensation at an external microwave frequency on the receiver side [5], but the extra fiber loss and cost for dispersion compensating fiber are the limiting factors of the above technique.

A technique based on a spectral inversion at the midpoint of the transmission length has been discussed in [6], but it offers a very low conversion rate, about −25 dB, and is expensive and complicated to implement in the communication systems.

An attempt has been made for the dispersion compensation by nonlinear optical phase conjugation (OPC) in which optical pulses distorted due to dispersion in the first fiber can be reshaped by following OPC and subsequent propagation through the second fiber with similar dispersive characteristics [7] and the effectiveness of this technique has been shown experimentally in [8].

A broadband, fiber-based technique for compensating dispersion in conventional SM fiber spans using higher-order modes near cutoff has been investigated in [9]. In this technique higher-order spatial modes in optical fibers exhibit large negative chromatic dispersion when operated near their cutoff wavelength. By using a spatial mode converter to selectively excite a higher-order mode in a specially designed multimode fiber, this dispersion can be used to compensate the positive dispersion in conventional SM fiber spans.

A method based on dispersion compensation using phase-sensitive optical amplifiers has been described in [10]. It can be implemented in both the positive and the negative GVD regions but demands phase tracking and locking to that of the propagating pulse.

In [11] a dispersion compensation scheme has been examined for lightwave systems based on the semiconductor Mach–Zehnder modulator.

A dispersion compensation technique based on propagation of signals through a specially designed fiber with large negative dispersion for the LP_{01} mode has been reported in [13].

Recently, work on dispersion compensation by differential time delay was reported in [14]. The proposed scheme was intended for high-bit-rate (> 10 Gbit/s) time division-multiplexed transmission, and it was shown that the transmitting distance could be enhanced by a factor of 4 in an ideal case and in the order of 2.5 under realistic conditions in a dispersive-limited system. In this technique, the analysis and implementation is based on the individual second-order (2OD) term of the propagation constant. Further, it has been proved that it is superior to common prechirp techniques.

An attempt has been made to compensate dispersion using the above technique by using other higher-order dispersion terms (3OD and 4OD) [15]. That paper presented the improved analysis of dispersion compensation using third- and fourth-order (3OD and 4OD) dispersion terms individually. RMS phase deviation, figure of merit, and dimension-free chirp parameter were evaluated and analyzed for ideal and realistic optical systems for 2OD, 3OD, and 4OD. Further, it was shown that the transmitting distance could be enhanced to fourfold, sixfold, and eightfold in an ideal case if the dispersion compensation is done by using 2OD, 3OD, and 4OD, respectively, and the results have also been obtained for realistic optical communication systems.

In this paper, we report our work on dispersion compensation by differential time delay using higher-order terms together. The combined effects of higher-order terms (2OD + 3OD) and (3OD + 4OD) have been analyzed when they are considered together instead of individual terms, as reported in [14, 15]. Further, RMS phase deviation, figure of merit, and dimension-free chirp parameter have been evaluated and a comparison is done with the results obtained in [14, 15].

2. ANALYSIS

We begin with the fundamental observation that dispersion in an optical fiber is a linear process when power levels are kept below those required at the onset of self-phase modulation or stimulated Raman scattering or nonlinear effects. Thus, the transmission characteristics of a single-mode fiber can be represented by a linear filter, which can be derived by Taylor expansion of the propagation constant β about the optical carrier frequency ω_c as mentioned in [14, 15],

$$\beta = \beta_c + \frac{1}{v_g}(\omega - \omega_c) - \frac{\lambda^2 D}{4\pi c}(\omega - \omega_c)^2 + \cdots, \tag{1}$$

where $v_g = \partial\omega/\partial\beta$ is the usual group velocity and D is the standard group delay dispersion parameter,

$$D = \frac{\partial}{\partial\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda}\frac{\partial^2\beta}{\partial\omega^2}. \tag{2}$$

For simplicity the propagation constant will be expressed as

$$\beta = \beta_c + D_1 \Delta\omega + \tfrac{1}{2} D_2 \Delta\omega^2 + \tfrac{1}{6} D_3 \Delta\omega^3 + \tfrac{1}{24} D_4 \Delta\omega^4 + \cdots, \quad (3)$$

where

$$D_1 = \frac{\partial \beta}{\partial \omega}, \qquad D_2 = \frac{\partial^2 \beta}{\partial \omega^2}, \qquad D_3 = \frac{\partial^3 \beta}{\partial \omega^3}, \qquad D_4 = \frac{\partial^4 \beta}{\partial \omega^4}, \ldots, \qquad D_n = \frac{\partial^n \beta}{\partial \omega^n};$$

$D_1, D_2, D_3, \ldots, D_n$ are first-, second-, third-, and nth-order dispersion terms, respectively, $(n = 1, 2, 3, \ldots)$ and $\Delta\omega = \omega - \omega_c$.

At the receiver the phase deviation may be expressed as

$$\phi = \phi_c - \beta L, \quad (4)$$

where L is the transmission distance in fiber.

Therefore,

$$\phi = \phi_c - \beta_c L - D_1 L \Delta\omega - \tfrac{1}{2} D_2 L \Delta\omega^2 - \tfrac{1}{6} D_3 L \Delta\omega^3 - \tfrac{1}{24} D_4 L \Delta\omega^4 - \cdots. \quad (5)$$

The first two terms on the right-hand side of Eq. (5) are constant phase contributions and the third term gives pure time delay. In the analysis the demands of absolute phase and time have been neglected. The phase contributions offered by the higher-order terms have been analyzed, when 2OD + 3OD and 3OD + 4OD terms of the propagation constant are taken together. Dispersion compensation is based on spectral separation of the signal in upper and lower sidebands and, after providing differential time delay, they are synchronously combined together for further transmission and detection. The upper and lower sidebands are separated through an integrated interferometer and time delay is provided in each arm. The total phase deviation, figure of merit, and dimension-free chirp parameter have been evaluated for ideal and realistic optical communication systems using higher-order dispersion terms together as mentioned below.

Case I: Using 2OD and 3OD terms together. In Eq. (5) neglecting the demands of absolute phase and time at the receiver side and considering the 2OD and 3OD terms together, the phase deviation can be expressed as

$$\phi_{23} = \tfrac{1}{2} D_2 L \Delta\omega^2 + \tfrac{1}{6} D_3 L \Delta\omega^3. \quad (6)$$

Further, the deviation in time arrival of different frequencies can be expressed as

$$\Delta t = -\frac{\partial \phi_{23}}{\partial \omega} = -D_2 L \Delta\omega - \frac{1}{2} D_3 L \Delta\omega^2, \quad (7)$$

where $\Delta t = t - t_c$ is the arrival time for the carrier frequency.

In order to compensate for the dispersion by differential time delay, the incoming signal is separated into two sidebands (upper and lower) using an integrated interferometer, and differential time delay in each arm of the interferometer can be expressed as $\pm C_{23}\omega_{cl}(D_2L/2 + D_3L\Delta\omega/4)$, where C_{23} is a dimensionless parameter and ω_{cl} is the angular clock frequency of the system. The phase deviation can now be expressed as

$$\phi_{23_{(total)}} = \frac{D_2L}{2}\left[\Delta\omega^2 \pm C_{23}\omega_{cl}\Delta\omega\right] + \frac{D_3L}{12}\left[2\Delta\omega^3 \pm 3C_{23}\omega_{cl}\Delta\omega^2\right], \quad (8)$$

where the plus sign is used for the lower sideband and the minus sign for the upper sideband. Measurement of *root mean square* (RMS) value of the phase deviation is the most common way to know the degradation in the dispersive systems. The phase deviation for the compensating device can be expressed as

$$\phi_{23_{RMS}} = \left[\int_{\omega-\omega_{cl}}^{\omega+\omega_{cl}} \frac{P(\Delta\omega)\left(\phi_{23_{(total)}}\right)^2}{2\omega_{cl}}\partial\omega\right]^{1/2} \quad (9)$$

$$\phi_{23_{RMS}} = \frac{L}{2\sqrt{\omega_{cl}}}\left[\int_0^{\omega_{cl}} P(\omega)\left\{D_2\left(\omega^2 - C_{23}\omega_{cl}\omega\right) + \frac{D_3}{6}\left(2\omega^3 - 3C_{23}\omega_{cl}\omega^2\right)\right\}^2 \partial\omega\right]^{1/2}. \quad (10)$$

At $\omega = \omega_{cl}$ and $P(\omega) = 1(\mathit{IdealCase})$,

$$\phi_{23_{RMS(total)}} = \frac{L}{2\sqrt{\omega_{cl}}}\left[D_2^2\omega_{cl}^5\left(\frac{1}{5} - \frac{C_{23}}{2} + \frac{C_{23}^2}{3}\right) + \frac{D_3^2\omega_{cl}^7}{36}\left(\frac{4}{7} - 2C_{23} + \frac{9C_{23}^2}{5}\right) + \frac{D_2D_3\omega_{cl}^6}{3}\left(\frac{1}{3} - C_{23} + \frac{3C_{23}^2}{4}\right)\right]^{1/2}. \quad (11)$$

From Eq. (2) $D_2 = -2\pi D/c\omega^2$. Therefore, $D_3 = -4\pi D/c\omega^3$ at standard group delay dispersion parameter D.

Hence, D_2 at $\omega = \omega_{cl}$ in terms of D_3 is given by $D_2 = \omega_{cl}D_3/2$ and Eq. (11) becomes

$$\phi_{23_{RMS(total)}} = \frac{D_3\omega_{cl}^3L}{2}\left[\frac{1}{4}\left(\frac{1}{5} - \frac{C_{23}}{2} + \frac{C_{23}^2}{3}\right) + \frac{1}{36}\left(\frac{4}{7} - 2C_{23} + \frac{9C_{23}^2}{5}\right) + \frac{1}{6}\left(\frac{1}{3} - C_{23} + \frac{3C_{23}^2}{4}\right)\right]^{1/2}. \quad (12)$$

Further, the figure of merit for the dispersion compensating device can be obtained at $C = 0$ and $C = C_{opt} = 0.68$ (obtained using minimization techniques as reported in [14, 15]). Therefore, the figure of merit is given as

$$G = \frac{\phi_{RMS_{(total)}}|_{C=0}}{\phi_{RMS_{(total)}}|_{C_{opt=0.68}}} = 5.03. \tag{13}$$

Thus, the figure of merit and optimum chirp parameter are found to be 5.03 and 0.68, respectively, using second- and third-order dispersion terms together when the weight function $P(\omega)$ has been set to 1 (ideal case). Hence, for an ideal case, this can be interpreted as, being possible to enhance the transmitting distance of an optical communication system by fivefold without degrading signal quality.

Case II: Using 3OD and 4OD terms together. Similarly, considering the 3OD and 4OD terms together, the phase deviation can now be expressed as

$$\phi_{34} = \tfrac{1}{6} D_3 L \Delta\omega^3 + \tfrac{1}{24} D_4 L \Delta\omega^4. \tag{14}$$

Further, the deviation in arrival time of different frequencies can be expressed as

$$\Delta t = -\frac{\partial \phi_{34}}{\partial \omega} = -\frac{1}{2} D_3 L \Delta\omega^2 - \frac{1}{6} D_4 L \Delta\omega^3, \tag{15}$$

where $\Delta t = t - t_c$ is the arrival time for the carrier frequency.

Similarly, in order to compensate for the dispersion by differential time delay, the incoming signal is separated into two sidebands (upper and lower) using an integrated interferometer, and differential time delay in each arm of the interferometer can be expressed as $\pm C_{34}\,\omega_{cl}(D_3 L \Delta\omega/4 + D_4 L \Delta\omega^2/12)$, where C_{34} is a dimensionless parameter and ω_{cl} is the angular clock frequency of the system. The phase deviation due to third- and fourth-order dispersion terms can now be expressed as

$$\phi_{34_{(total)}} = \frac{D_3 L}{12}\left[2\Delta\omega^3 \pm 3C_{34}\,\omega_{cl}\Delta\omega^2\right] + \frac{D_4 L}{24}\left[\Delta\omega^4 \pm 2C_{34}\,\omega_{cl}\Delta\omega^3\right]. \tag{16}$$

Measurement of the RMS value of the phase deviation is the most common way to know the degradation in the dispersive systems. Therefore, the RMS phase deviation for the compensating device can be expressed as

$$\phi_{34_{RMS}} = \frac{L}{24\sqrt{\omega_{cl}}}\left[\int_0^{\omega_{cl}} P(\omega)\left\{2D_3\left(2\omega^3 - 3C_{34}\,\omega_{cl}\,\omega^2\right) + D_4\left(\omega^4 - 2C_{34}\,\omega_{cl}\,\omega^3\right)\right\}^2 \partial\omega\right]^{1/2}. \tag{17}$$

At $\omega = \omega_{cl}$ and $P(\omega) = 1(\mathit{IdealCase})$,

$$\phi_{34_{RMS_{(total)}}} = \frac{D_4\,\omega_{cl}^4 L}{24\sqrt{\omega_{cl}}}\left[4D_4^2\omega_{cl}^7\left(\frac{4}{7} - 2C_{34} + \frac{9C_{34}^2}{5}\right) + D_4^2\omega_{cl}^9\left(\frac{1}{9} - \frac{C_{34}}{2} + \frac{4C_{34}^2}{7}\right) + 4D_3 D_4\,\omega_{cl}^8\left(\frac{1}{4} - C_{34} + C_{34}^2\right)\right]^{1/2}. \tag{18}$$

Similarly, by putting $D_3 = D_4 \omega_{cl}$ and obtaining Eq. (18) in terms of D_4 as below,

$$\phi_{34_{RMS_{(total)}}} = \frac{D_4 \omega_{cl}^4 L}{12} \left[\left(\frac{4}{7} - 2C_{34} + \frac{9C_{34}^2}{5} \right) + \frac{1}{4} \left(\frac{1}{9} - \frac{C_{34}}{2} + \frac{4C_{34}^2}{7} \right) + \left(\frac{1}{4} - C_{34} + C_{34}^2 \right) \right]^{1/2}. \quad (19)$$

Similarly, the value of C_{34} shall now be chosen so that the RMS value is minimized and is found out to be 0.53. Further, the figure of merit for the dispersion compensating device can now be obtained as

$$G = \frac{\phi_{RMS_{(total)}}|_{C=0}}{\phi_{RMS_{(total)}}|_{C_{opt=0.53}}} = 6.58. \quad (20)$$

The figure of merit and optimum chirp parameter are found to be 6.58 and 0.53, respectively, using third- and fourth-order dispersion terms together when the weight function is $P(\omega)$ has been set to 1 (ideal case). Hence, for an ideal case, this can be interpreted as, being possible to enhance the transmitting distance of an optical communication system by 6.5-fold without degrading signal quality.

3. RESULTS AND DISCUSSION

In Eqs. (10) and (17), weight function in the integral corresponds to the signal spectrum being involved and it was anticipated that the weight function is 1, which means that we would require a sinc as the pulse envelope in the system, but the realistic choice for $P(\omega)$ is the continuous function where $P(0) = 1$ and $P(\pm\omega_{cl}) = 0$. Therefore, it is very important to obtain the C and G values for the realistic (weight functions) optical communication systems. Table 1 shows the values C and G for the ideal and realistic weight functions. Similar weight functions are chosen as reported in [14, 15] to know the comparison for C and G values. In [15] it is

TABLE 1

	2ODT		3ODT		4ODT		2ODT + 3ODT		3ODT + 4ODT	
$P(\omega)$	C	G	C	G	C	G	C	G	C	G
1(*IdealCase*)	0.75	4.0	0.55	6.0	0.43	8.0	0.67	5.0	0.53	6.58
$1 - \left\lvert \frac{\omega}{\omega_{cl}} \right\rvert$	0.60	3.2	0.48	4.6	0.39	6.1	0.56	4.02	0.46	4.96
$\cos^2\left(\frac{\pi\omega}{2\omega_{cl}}\right)$	0.53	3.0	0.43	4.3	0.35	5.9	0.50	3.52	0.42	4.54
$\frac{10}{9}\cos^2\left(\frac{\pi\omega}{2\omega_{cl}}\right) - \frac{1}{9}\cos^2\left(\frac{3\pi\omega}{2\omega_{cl}}\right)$	0.48	3.1	0.39	4.2	0.32	6.1	0.45	3.54	0.38	4.30

described that the C value is normally within the interval [0.6–0.48], [0.48–0.39], and [0.39–0.32] and the G value is normally within the interval [3.2–3.0], [4.6–4.2], and [6.1–5.9] if obtained for 2OD, 3OD, and 4OD terms individually for the realistic weight functions (signal spectrums).

This paper presents that the C value is normally within the interval [0.56–0.45] and [0.46–0.38] and the G value is normally within the interval [4.02–3.54] and [4.96–4.30] if obtained for 2OD + 3OD and 3OD + 4OD terms together for the realistic weight functions. Here, the results show that C and G values are more accurate (because of combined effects of dispersion terms) and within the values obtained in [15] using individual dispersion terms. Similarly, the figure of merit increases, whereas the dimension-free chirp parameter decreases. Hence, it is possible to enhance the transmitting distance of an optical communication system without degrading signal quality when the dispersion compensating device uses higher-order dispersion terms. Practically, it can be implemented using an integrated interferometer as mentioned in [14, 15]. The method proposed would be most easy to implement as very high bit rates, i.e., 20 or 40 Gb/s, since the power spectrum in that case is wider. The wider the modulated power spectrum, the easier it will be to implement low-loss, sharp, polarization-independent optical fibers [14].

4. CONCLUSION

This paper presents the detailed theoretical analysis for the dispersion compensation by differential time delay using higher-order (2OD + 3OD and 3OD + 4OD) dispersion terms. Very accurate results are obtained because of combined effects of higher-order dispersion terms for the ideal and realistic weight functions. Further, it has been shown that the dispersionless propagation length can be enhanced over 5 and 6.5 times longer than the usual dispersion length of fiber in the ideal, 3.5 and 4.5, times in realistic optical communication systems if compensation is performed using 2OD + 3OD and 3OD + 4OD terms, respectively.

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